# On Input-Output Analyses With Incomplete Data 

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The rising powers of personal computers and software increase the ease with which one can make input-output projections and analyses. Unfortunately, individual Canadian researchers who use Canadian data can only achieve results of questionable accuracy.

Of all the countries in the world, Canada is perhaps the only one where the central statistical office -- in this case Statistics Canada -- publishes unbalanced input-output data. The data are unbalanced in the sense that supply does not equal demand for several commodities and industries.

The main reason for lack of balance in published input-output data is the suppression of confidential cell values in the data matrices. Small aggregation, 16 -industry tables, do not suffer from this defect. Confidentiality is a concern in the larger, more disaggregated tables, which also are the more useful ones.

There are also significant rounding errors in special tabulations prepared by Statistics Canada on request of customers. Values in special aggregations are rounded to millions of dollars so that, in effect, all cells containing less than $\$ 500,000$ are suppressed by rounding down to zero. Other cells under $\$ 10,000,000$ also suffer relatively large rounding errors of more than 5 per cent.

Lack of balance in the data leads to inaccuracies in projections and other computational results. For many years, the solution was to purchase from Statistics Canada not the data, but the results of computations made by Statistics Canada with its access to complete data. This solution costs time and money that researchers must balance with the inaccuracy of their own, cheaper results.

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[^0]This note presents a new method for achievement of accurate and nearly accurate results with public use data. The solution is an application of the "dummy industry" technique already in use. ${ }^{1}$ The new solution is peculiar to a Canadian data problem and therefore not yet found in the international inputoutput literature. The statement of the solution follows a brief restatement of the problem.

Let $q$ be a vector of $m$ commodity outputs,
$g$ be a vector of $n$ industry outputs,
$V$ be a $n \times m$ matrix of industrial commodity outputs,
$U$ be a $m \times n$ matrix of commodity inputs to industries,
$e$ be a $m \times l$ vector of final demands for commodities,
$y$ be a $n \times 1$ vector of value added by industry,
then, using primes to denote transposition, carets for diagonalization and $i$ for a unit vector,

$$
\begin{align*}
& g=V i  \tag{1}\\
& g=U^{\prime} i+y  \tag{2}\\
& q=V^{\prime} i  \tag{3}\\
& q=U i+e \tag{4}
\end{align*}
$$

Defining technical production and market share coefficients by

$$
\begin{aligned}
& B=U \hat{g}^{-1} \\
& D=V \hat{q}^{-1}
\end{aligned}
$$

one obtains after some manipulation

$$
\begin{equation*}
[I-B D] q=e \tag{5}
\end{equation*}
$$

and $[I-D B] g=D e$

Equations (5) and (6) can be solved for $q$ and $g$ by matrix inversion assuming (1)-(4) hold. Excessive rounding and suppression of confidential values mean that public use data do not meet conditions (1)-(4). Consequently, users of such data obtain incorrect values of $B$ and $D$ and erroneous solutions for $q$ and $g$. The problem is not confined to the output determination models in (5) and (6). It, of course, affects also the dual price models.

1. On the "dummy industries" see Dominion Bureau of Statistics, The input-output structure of the Canadian economy, 1961, vol. I, August 1969, page 121; and Statistics Canada, The input-output structure of the Canadian economy, 1971-80, catalogue No. 15-201E, page 23.

The problem is not that equations (5) and (6) are invalid but that the assumptions from which they were derived are violated. The solution is then to re-balance the tables so that the assumptions can be maintained. A further problem is that $V$ and $U$ cannot be re-balanced individually because different data are suppressed from each. What is required is a simultaneous re-balancing that restores the equalities between supplies and demands for commodities and industry outputs.

The solution is to work simultaneously in commodity and industry space augmented by a dummy activity that balances the public use tables. Analogous to three dummy activities and dummy commodities already in existing inputoutput data and models, the new dummy activity uses goods of unknown origin to produce a fictitious commodity used by all other activities in place of unknown, alternative inputs. The inputs to the new dummy activity and the uses of its output are residuals that balance the public use data. The residuals can be computed using any available control totals or without benefit of such control totals. The latter alternative is discussed first.

Let the public use data be arranged in a square, partitioned, national accounting matrix:

$$
M^{*}=\left[\begin{array}{ccc}
0 & V & 0 \\
U & 0 & e \\
y^{\prime} & 0 & 0
\end{array}\right]
$$

where $q^{*}=M^{*} i ; g^{*}=M^{*} i$ and asterisks denote unbalanced data.
Balancing the national accounts requires

$$
\begin{equation*}
q_{i}^{*}=g_{i}^{*} \quad \forall i \in(1 \leq i \leq n+m) \tag{7}
\end{equation*}
$$

Public use data do not satisfy (7). One can thus create two dummy vectors $r$ and $s$ whose elements are given by

$$
\begin{aligned}
& r_{i}=\max \left(g_{i}^{*}-q_{i}^{*}, 0\right) \quad \forall i \\
& s_{j}=\max \left(q_{j}^{*}-g_{j}^{*}, 0\right) \quad \forall j
\end{aligned}
$$

The national accounts balanced by $r$ and $s$ are now

$$
M^{* *}=\left[\begin{array}{cccc}
0 & V & r^{1} & 0 \\
U & 0 & r^{2} & e \\
s^{\prime} & s^{2^{\prime}} & 0 & 0 \\
y^{\prime} & 0 & 0 & 0
\end{array}\right]
$$

where a superscript 1 denotes residuals that balance industries; a superscript 2 denotes residuals that balance commodities. $V$ and $U$ cannot be balanced separ-
ately because $r^{\prime} i \neq s^{\prime} i$ and $r^{2} i \neq s^{2} i$; only $M^{*}$ can be balanced without disclosure of confidential data.

The new matrix is such that

$$
q_{i}^{* *}=g_{i}^{* *} \quad \forall i \in(1 \leq i \leq n+m)
$$


and so the new technical coefficients are

$$
A=M^{* *}\left(\hat{g}^{* *}\right)^{-1}
$$

and, after deleting final demand and value added from $M^{* *}$ and $A$ to close the model,

$$
\begin{equation*}
g^{* *}=[I-A]^{-1} e^{* *}=q^{* *} \tag{8}
\end{equation*}
$$

is the simultaneous solution in both industry and commodity spaces. ${ }^{2}$
The model in (8) produces much improved results. Some errors in projected industry and commodity outputs vanish. Other errors are drastically reduced. Remaining errors are confined to industries and commodities that suffer most from data suppression, such as tires and tubes, and processed, unmanufactured tobacco. National, medium level tables for the year 1988 were used to provide examples of alternative output projections. Industry and commodity outputs required to meet actual final demands were computed. The results are shown in Table $1 .{ }^{3}$ Column (1) of Table 1 shows the control totals $g$ and $q$ in millions of dollars at 1988 prices. Column (2) shows the outputs projected from the actual final demand $e$ with equations (5) and (6). The projection error is shown in column (3) as a per cent of column (1). Column

[^1](4) shows the outputs projected with equation (8). The reduced percentage errors are shown in column (5). For example, the error in projected output of the nickel products commodity was reduced from 100 per cent in column (3) to only 2.7 per cent in column (5). This reduction was achieved by creation of a dummy residual activity whose inputs and outputs were estimated without benefit of control totals. The accuracy of results obtained by this method varies according to the severity of data suppression. Data suppression increases in severity as increased disaggregation unveils more and more confidential cell values. Data suppression varies also from year to year in the national tables. Data suppression also varies among provincial tables; it presents a larger problem in tables for the smaller provinces. ${ }^{4}$

Exact results are obtained when control totals are used to fill dummy vectors; that is, when

$$
r=\left[\begin{array}{l}
g \\
q
\end{array}\right]-q^{*} ; \quad s=\left[\begin{array}{l}
g \\
q
\end{array}\right]-g^{*}
$$

The vectors $g$ and $q$ are published to the nearest $\$ 100,000$ at the 50 -industry, 92 -commodity, medium level of aggregation. No results are shown for this case as the errors are all zero. ${ }^{5}$

Users can usually obtain at least $g$ at other aggregation levels. Thus, a combination of the two approaches to $r$ and $s$ is possible. Virtually exact industry output projections using residuals based on published industry output figures are shown in columns (6) and (7) of Table 1. These last results were obtained by letting

$$
\begin{aligned}
r^{1} & =g-V i \\
s^{1} & =g-U^{\prime} i-y \\
r_{i}^{2} & =\max \left(v_{i}-u_{i}, 0\right) \quad \forall i \\
s_{j}^{2} & =\max \left(u_{j}-v_{j}, 0\right) \quad \forall j \\
\text { ere } & \\
v & =V^{\prime} i \\
u & =U i+e
\end{aligned}
$$

where

Commodity output projections remained inexact because residual commodity flows were not based on control totals. Again, the inaccuracy of the commodity output projections depends on the case. However, the exact industry
4. That accuracy varies in relation to disaggregation and economic size was also noted by one of the referees.
5. Exact results were obtained also in the author's application of a larger, 73-industry, 116commodity price model in "The Effect of Indirect Tax, Wage, and Regulated Price Changes on the Competitiveness of Canadian Exports," a paper prepared for Queens'-University of Ottawa Economic Projects, May 1993.
projections are sufficient for attainment of exact projections of employment and other variables proportional to industry output.

In conclusion, data suppression and rounding are no longer impediments to input-output modelling with public use data, least of all when sufficient control totals are available.

TABLE 1 Errors in Projections With Incomplete, 1988 Input-Output Data

|  | CORRECT value (1) | CONVENTIONAL PRONECTION <br> (2) | $\begin{array}{\|c\|} \hline \text { ERROR } \\ \text { in \% } \\ (3) \\ \hline \end{array}$ | NEW <br> PROJECTION <br> (4) | $\substack{\text { ERROR } \\ \text { in \% } \\ \text { (5) }}$ |  | ERROR in \% (7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | industry output |  |  |  |  |  |  |
| Agriculure | 24886.7 | 24497.4 | -0.36 | 24886.7 |  | 24586.7 |  |
| Fisting, bunting and lrapping | 1700.7 | $15 \% 6.7$ | -6.12 | 1700.7 |  | 1700.7 |  |
| Lugeing and forestry | 8702.4 | 8643.8 | 0.67 | 8702.4 |  | 8702.4 |  |
| Minuıg | 14562,0 | 13948.8 | 4.21 | 14552.8 | -0.06 | 14562.0 |  |
| Cructe petroleum and natural gas extration | 17385.0 | 17320.9 | -0.37 | 17385.1 | 0.00 | 17385.1 | 0.00 |
| Quarrics | 1384.8 | 1365.0 | -1.43 | 1384.9 | 0.01 | 1384.9 | 0.01 |
| Services io miuing | 4283.4 | 4261.3 | -0.52 | 4283.4 |  | 4283.4 |  |
| Fuad | 38554.1 | 38475.9 | -0.20 | 38536.5 | -0.05 | 38.55 .1 |  |
| Beverages | 6333.3 | 6353.7 | 0.48 | 6323.2 | 0.00 | 6323.2 | 0.00 |
| Totacco | 1749.5 | 1468.0 | -16.09 | 1476.5 | -15.60 | 1749.5 |  |
| Rubber | 2929.1 | 2528.7 | 13.67 | 2801.9 | 4.34 | 2929.1 |  |
| Plastic | 6043.4 | 5877.3 | -2.75 | 6015.9 | 0.46 | 8043.4 |  |
| Leather | 1316.3 | 1099.9 | -16.44 | 1252.8 | 4.82 | 1316.3 |  |
| Textiles | 6711.0 | 6740.8 | 0.44 | 6553.6 | -2.35 | 6711.0 |  |
| Clowhing | 6837.9 | 6798.5 | -0.58 | 6814.2 | 0.35 | 6837.9 |  |
| wood | 15503.8 | 15433.4 | -0.45 | 15501.2 | -0.02 | 15503.8 |  |
| Furniture | 4672.9 | 4574.2 | -2.11 | 4653.3 | 0.42 | 4672.9 |  |
| Paper | 26019.6 | 25431.4 | -2.26 | 25865.4 | -0.59 | 26019.6 |  |
| Prining and publisting | 12842,2 | 12619.9 | -1.73 | 12826.3 | -0.12 | 12842.2 |  |
| Primary mctal products | 28336.0 | 23738.6 | -16.22 | 28304.0 | 0.11 | 28336.0 |  |
| Metal fabricating | 18339.2 | 17316.4 | -5.58 | 18289.1 | 0.27 | 18339.2 |  |
| Mactivery | 10401.8 | 10515.6 | 1.09 | 10383.5 | 0.18 | 10401.8 |  |
| Transport equipment | 53885.4 | 54292.9 | 0.76 | 53245.5 | -1.19 | 53885.4 |  |
| Electical mrodicts | 19236.4 | 19385.4 | 0.77 | 1990.4 | 0.24 | 19236.4 |  |
| Non-metallic prodicis | 8110.1 | 8996.8 | -0.16 | 8086.1 | -0.30 | 810.1 |  |
| Refined petroleum and coal prochuts | 14180.7 | 14098.0 | -0.58 | 14178.0 | -0.02 | 14180.7 |  |
| Chemixals | 29572.5 | 23320.2 | -1.07 | 23515.7 | 0.24 | 23572.5 |  |
| Other mamufacuring | 64\%.3 | 6152.8 | -5.29 | 6394.5 | -1.57 | 6496.3 |  |
| Construction | 90268.9 | 90220.1 | $-0.05$ | 90268.9 |  | 90268.9 |  |
| Transport | 38217.7 | 38064.5 | 0.40 | 38219.8 | 0.00 | 38217.7 |  |
| Pipeline transport | 2926.0 | 2906.6 | 0.66 | 2926.0 |  | 2926.0 |  |
| Storage | 1156.2 | 1151.4 | -0.42 | 1156.3 | 0.01 | 1156.3 | 0.01 |
| Communications | 18830.3 | 19811.1 | -0.10 | 19830.3 |  | 19830.3 |  |
| Oherer utilites | 21601.7 | 21347.0 | -1.18 | 21601.7 |  | 21601.7 |  |
| Wholesale trade | 42178.9 | 42024.5 | -0.37 | 42178.8 | 0.00 | 42178.9 |  |
| Retail trade | 58876.3 | 50847.8 | 0.06 | 50876.4 | 0.00 | 50876.4 | 0.00 |
| Finance and real estare | 72105.3 | 71981.0 | -0.17 | 72105.3 |  | 72105.3 |  |
| Insurance | 10167.1 | 10149.6 | -0.17 | 10167.2 | 0.00 | 10167.2 | 0.00 |
| Royalies | 3473.5 | 3467.9 | -0.17 | 3473.5 |  | 3473.5 |  |
| Owner-accupied dwellings | 44770.4 | 44770.4 |  | 44770.5 | 0.00 | 44770.5 | 0.00 |
| Business services | 31367.2 | 31354.8 | -0.04 | 31367.3 | 0.00 | 31367.3 | 0.00 |
| Edication | 1626.7 | 1626.7 |  | 1627.1 | 0.02 | 1627.1 | 0.02 |
| Health services | 14503.8 | 14503.1 | 0.00 | 14504.0 | 0.00 | 14504.0 | 0.00 |
| Accommotation and food | 23697.9 | 23689.0 | $-0.04$ | 23697.9 |  | 23697.9 |  |
| Anusementand recreation | 8136.6 | 8134.9 | -0.02 | 8136.7 | 0.00 | 8136.7 | 0.00 |


|  | correct value (1) | CONVENTIONAL prosection <br> (2) | $\begin{array}{\|c\|} \hline \text { ERROR } \\ \text { in \% } \\ \text { (3) } \end{array}$ | NEW <br> PROFCTION <br> (4) | $\begin{array}{\|c} \text { ERROR } \\ \text { in } 8 \\ (5) \end{array}$ | LAST Prajection (6) | $\begin{gathered} \text { ERROR } \\ \text { in } \% \\ \text { i } \% \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Personal and bouschold services | ${ }^{6725.9}$ | 674.1 | 0.18 | 6726.0 | 0.00 | 6726.0 | 0.00 |
| Ohber services | 9887.9 | 9872.0 | -0.16 | 9887.8 | 0.00 | 9887.8 | 0.00 |
| Supplies | 24398.9 | 24170.7 | -0.94 | 24398.9 |  | 24398.9 |  |
| Travel, adverising and promotion | 20245.4 | 20205.0 | 0.20 | 20245 | 0.00 | 20245.5 | 0.00 |
| Trastsport margins | 17081.4 | 16958.8 | -0.72 | 17081.4 |  | 17081.4 |  |
| total industry output | 939910.4 | 92992.8 | -1.06 | 938048.9 | -0.20 | 939911.9 | 0.00 |
|  | COMMODITY OUTPUT |  |  |  |  |  |  |
| grains | 5158.6 | 5152.0 | -0.13 | 5188.7 | 0.00 | 5158.7 | 0.00 |
| live animals | 6590.3 | 6898.7 | 0.15 | 6909.3 |  | 6909.3 |  |
| other agricultural products | 11484.3 | 11413.0 | 0.62 | 11484.4 | 0.00 | 11484.4 | 0.00 |
| FORESTRY PRODUCTS | 8338.9 | 8282.2 | -0.68 | 8338.8 | 0.00 | 8338.8 | 0.00 |
| fish landings | 1613.5 | 1610.2 | -0.20 | 1613.5 |  | 1613.5 |  |
| hunting and trapping products | 65.1 | -35.8 | -154.99 | 64.9 | -0.31 | 64.9 | -0.31 |
| iron ores and concentrates | 1211.7 | 1130.9 | -6.67 | 121.7 |  | 121.7 |  |
| other metallic ores and cONCENTRATES | 11099.3 | 9307.7 | -16.14 | 10229.4 | -7.84 | 10229.4 | -7.84 |
| coal | 1545.3 | 1447.9 | -6.30 | 1544.9 | -0.03 | 154.9 | -0.03 |
| crude mineral oils | 9890.7 | 9855.1 | -0.26 | 9890.7 |  | 9890.7 |  |
| natural gas | 584.1 | 5854.3 | 0.68 | 589.1 |  | 5894.1 |  |
| non-metallic minerals | 2663.8 | 2632.0 | -4.19 | 2651.0 | 0.48 | 2651.0 | 0.48 |
| Services incidental to mining | 4361.5 | 4339.6 | -0.50 | 4361.5 |  | 4361.5 |  |
| meat products | 11596.5 | 11485.5 | -0.9 | 11535.3 | 0.53 | 11535.3 | 0.53 |
| dalry products | 6845.9 | 6840.9 | -0.07 | 6845.9 |  | 6845.9 |  |
| fish products | 2789.4 | 2788.5 | -0.03 | 2789.4 |  | 2789.4 |  |
| FRUITS AND VEGETABLES PREPARATIONS | 3865.7 | 3863.4 | 0.06 | 3865.3 | -0.01 | 3865.3 | 0.01 |
| feeds | 3467.0 | 3457.2 | -0.28 | 3456.9 | 0.00 | 3466.9 | 0.00 |
| FLOUR. WhEAT, MEAL \& OTHER cereals | 839.5 | 836.3 | 0.38 | 839.5 |  | 839.5 |  |
| bREAKFAST CEREAL \& BAKERY PRODUCTS | 3387.5 | 3385.7 | -0.0s | 3387.6 | 0.00 | 3387.6 | 0.00 |
| SUGAR | 502.9 | 503.8 | 0.18 | 502.7 | -0.04 | 502.7 | 0.04 |
| miscellaneous food products | 5246.8 | 524.7 | 0.10 | 5242.4 | -0.08 | 5242.4 | 0.08 |
| Soft drinks | 2048.0 | 2054.9 | 0.34 | 2048.0 |  | 2048.0 |  |
| alcoholic beverages | 3789.2 | 378.9 | 0.02 | 3789.2 |  | 3789.2 |  |
| tobacco processed UNMANUFACTURED | 247.3 |  | -100.00 | 17.5 | -92. 22 | 17.9 | -92.92 |
| cigarettes and tobacco mfg. | 1448.3 | 1448.3 |  | 1448.3 |  | 1448.3 |  |
| TIRES AND TUBES | n/a |  | 7/ | 132.0 | n/2 | 132.0 | n/a |
| Other rubber products | 2244.0 | 2066.7 | -6.56 | 2215.4 | -1.27 | 2215.4 | -1.27 |
| PLASTIC Fabricated products | 4663.7 | 4545.7 | -2.53 | 4574.8 | -1.91 | 4574.8 | -4.91 |
| leather and leather products | 1136.4 | \$64.6 | -15.12 | 1117.6 | -1.65 | 1117.6 | -1.65 |
| yarns and man made fibres | 1463.6 | 1501.0 | 2.56 | 1444.3 | -1.32 | 1444.3 | -1.32 |
| fabrics | 2070.3 | 2082.0 | 0.57 | 2041.1 | -1.41 | 2041.1 | -1.41 |
| Other textile products | 2888.0 | 2827.9 | -1.05 | 2819.9 | -1.33 | 2819.9 | -1.33 |
| Hosiery and knitted wear | 1203.8 | 1203.1 | 0.06 | 1203:1 | -0.06 | 1203.1 | 0.06 |
| clothing and accessories | 5558.2 | 5443.2 | -2.07 | 5455.5 | -1.85 | 5455.5 | -1.85 |
| LUMBER AND TIMBER | 7648.4 | 7586.9 | $-0.80$ | 7594.2 | -0.71 | 7594.2 | -0.71 |
| VENEER AND Plywood | 1072.6 | 1063.0 | -0.00 | 1065.9 | -0.62 | 1065.9 | 0.62 |
| other wood fabricated material | 6830.1 | 6779.0 | -0.60 | 6819.3 | -0.01 | 6819.3 | -0.01 |
| furnture and fixtures | 4881.9 | 4869.6 | -0.25 | 4873.6 | 0.17 | 4873.6 | -0.17 |
| pulp | 7431.7 | 734.1 | -1.04 | 2431.7 |  | 7431.7 |  |
| NEWSPRINT \& OTHER PAPER STOCK | 12233.1 | 12058.5 | -1.43 | 12222.9 | -0.08 | 12722.9 | 0.08 |
| PAPER PRODUCTS | 7255.1 | 7222.7 | -0.45 | 7255.3 | 0.00 | 7255.3 | 0.00 |
| Printing and publishing | 8886.2 | 8752.9 | -1.61 | 8894.7 | 0.02 | 8894.7 | -0.02 |
| advertising. print media | 3230.7 | 323.4 | -0.23 | 3230.7 |  | 3230.7 |  |


|  | CORRECT value <br> (1) | CONVENTIONAL pronection <br> (2) | $\begin{gathered} \hline \text { ERROR } \\ \text { in }{ }_{2} \\ \text { (3) } \end{gathered}$ | NEW <br> PROJECTION <br> (4) | $\begin{gathered} \hline \text { ERROR } \\ \text { in } \% \\ (5) \end{gathered}$ | LAST PROJECTION (6) | $\begin{gathered} \hline \text { ERROR } \\ \mathbf{c} \begin{array}{c} \mathrm{in} \% \\ (7) \end{array} \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| iron and steel products | 11926.8 | 11568.0 | -3.01 | 11919.2 | -0.06 | 11919.2 | -0.06 |
| aluminum products | 6201.7 | 5874.3 | -5.28 | 6163.6 | -0.61 | 6163.6 | -0.61 |
| COPPER \& COPPER ALLOY PRODUCTS | 2421.2 | 2356.0 | -2.69 | 2416.1 | -0.21 | 2416.1 | -0.21 |
| nickel products | 2557.9 |  | -100.00 | 2489.0 | -2.69 | 2489.0 | -2.69 |
| OTHER NON FERROUS METAL PRODUCTS | 3319.1 | 3108.1 | -6.36 | 3313.1 | -0.18 | 3313.1 | -0.18 |
| bollers. tanks and plates | 1078.0 | 1054.3 | 2.20 | 1065.1 | -1.20 | 1065.1 | -1.20 |
| fabricated structural metal products | 4038.0 | 4000.8 | -0.92 | 4020.8 | -0.43 | 4020.8 | $\bigcirc .0$ |
| other metal fabricated products | 11663.3 | 10021.4 | -14.08 | 11596.0 | -0.58 | 11596.0 | -0.58 |
| agricultural machinery | 995.0 | 982.7 | -0.23 | 994. | -0.06 | 994.4 | -0.06 |
| other industrial machinery | 12653.1 | 12646.0 | -0.06 | 12643.6 | -0.08 | 12643.6 | -0.08 |
| motor vehicles | 29587.3 | 29582.3 | -0.02 | 29578.4 | -0.03 | 29578.4 | -0.03 |
| motor vehicle parts | 15689.9 | 16564.2 | 5.57 | 15644.0 | -0.23 | 15644.0 | -0.23 |
| OTHER TRANSPORT EQUTPMENT | 7682.8 | 7748.1 | 0.85 | 7677.2 | -0.07 | 7677.2 | -0.07 |
| appliances and receivers. household | 2531.0 | 2345.1 | 0.56 | 2527.9 | 0.12 | 2527.9 | -0.12 |
| OTHER ELECTRICAL PRODUCTS | 13070.4 | 13124.2 | 0.41 | 13063.8 | 0.05 | 13063.8 | -0.0s |
| Cement and Concrete products | 3987.1 | 3980.4 | -0.17 | 3980.3 | -0.17 | 3980.3 | -0.17 |
| other non-metallic mineral PRODUCTS | 3715.4 | 3689.2 | -0.71 | 3714.0 | -0.04 | 3714.0 | -0.04 |
| gasoline and fuel oil | 10555.1 | 10510.0 | -0.43 | 10554.8 | 0.00 | 10544.8 | 0.00 |
| other Petroleum and coal PRODUCTS | 5052.2 | 4996.7 | -1.10 | 5052.0 | 0.00 | 5052.0 | 0.00 |
| industrial chemicals | 11792.2 | 11625.5 | -1.41 | 11783.2 | -0.08 | 11783.2 | -0.08 |
| fertilizers | 2274.2 | 2267.0 | -0.32 | 2272.7 | -0.07 | 2272.7 | 0.07 |
| pharmaceuticals | 2978.4 | 2972.3 | -0.20 | 2977.9 | -0.02 | 2977.9 | -0.02 |
| Other chemical products | 7566.2 | 7524.1 | -0.56 | 7563.3 | -0.04 | 7563.3 | 0.04 |
| SCIENTIFIC EQUIPMENT | 2821.5 | 2561.5 | -9.21 | 2770.2 | -1.82 | 2770.2 | -1.82 |
| OTHER MANUFACTURED PRODUCTS | 4778.3 | 4648.1 | 2.72 | 4665.0 | -2.37 | 4655.0 | -2.37 |
| residential construction | 34634.8 | 34634.8 |  | 34634.8 |  | 34634.8 |  |
| non-residential construction | 42415.0 | 42415.0 |  | 42415.0 |  | 42415.0 |  |
| repair construction | 12729.3 | 12883.1 | 0.36 | 12729.3 |  | 12729.3 |  |
| pipeline transportation | 3042.6 | 302.1 | 0.67 | 3042.6 |  | 3042.6 |  |
| transportation and storage | 38318.5 | 38156.0 | -0.42 | 38318.5 |  | 38318.5 |  |
| radio and television broadcasting | 3366.4 | 3361.7 | 0.14 | 3366.4 |  | 3366.4 |  |
| telephone and telegraph | 12428.0 | 124070 | 0.17 | 12428.1 | 0.00 | 12428.1 | 0.00 |
| postal services | 3211.9 | 3205.5 | -0.20 | 3211.9 |  | 3211.9 |  |
| electric power | 18062.6 | 17914.5 | -0.82 | 18062.7 | 0.00 | 18062.7 | 0.00 |
| OTHER UTILITIES | 2946.8 | 2845.9 | -3.42 | 2942.8 | -0. 14 | 2942.8 | 0.14 |
| wholesale margins | 43663.2 | 43475.6 | -0.43 | 43663.2 |  | 43663.2 |  |
| retail margins | 42585.0 | 42571.4 | 0.03 | 42585.2 | 0.00 | 42585.2 | 0.00 |
| IMPUTED RENT OWNER OCPD. dwellings | 44770.4 | 44770.4 |  | 44770.4 |  | 44770.4 |  |
| OTHER FINANCE, INSURANCE, REAL ESTATE | 8424.12 | 84099.8 | -0.18 | 84244.3 | 0.00 | 84244.3 | 0.00 |
| business services | 33353.0 | 33332.2 | -0.06 | 33353.3 | 0.00 | 33353.3 | 0.00 |
| education services | 1626.7 | 1626.7 |  | 1626.7 |  | 1626.7 |  |
| health services | 14316.3 | 14316.2 | 0.00 | 14316.3 |  | 14316.3 |  |
| Amusement \& RECREATION SERVICES | 7445.7 | 7444.4 | -0.02 | 7445.8 | 0.00 | 7445.8 | 0.00 |
| accommodation and food services | 24489.7 | 24480.7 | -0.04 | 24490.0 | 0.00 | 24490.0 | 0.00 |
| Other personal \& Misc. Services | 32620.7 | 32557.6 | -0.19 | 32620.8 | 0.00 | 32620.8 | . 00 |
| transportation margins | 17881.4 | 16960.7 | -0.71 | 17081.6 | 0.00 | 17881.6 | 0.00 |
| operating, office, lab \& food | 24398.9 | 4177 | -0.91 | 99.3 | 0.00 | 24399.3 | 0.00 |
| travel, advertising \& Promotion | 20245.4 | 20199.9 | -0.22 | 20245.8 | 0.00 | 20245.8 | 0.00 |
| total commodity output | 939910.4 | 92977.2 | -1.08 | 937973.3 | -0.21 | 937973.3 | -0.21 |
| residual | n/a | n/2 | n/2 | 8009.2 | n/2 | 9872.2 | n/2 |

Note: A blank denotes an exact zero. n/a: either not available or not applicable.


[^0]:    - Canadian Joumal of Regional Science/Revue canadienne des sciences régionales, XVI: 1 (Spring/printemps 1993), 115-122.
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[^1]:    2. The models in (5), (6) and (8) correspond to the open model without leakages presented in Statistics Canada, The input-output structure of the Canadian economy, 1961-81, Catalogue 15-510, December 1987. The simultaneous approach in (8) simplifies the algebra of the models with leakages. Matrix $M$ can be augmented by three rows and three columns for inventory change, govermment and the rest of the world. Expenditures on additions to inventories, govemment consumption and exports go in the columns. Negative columns in Statistics Canada's final demand tables are transposed and changed in sign. They represent receipts by withdrawals from inventory, government sales of goods and services, and imports that go in the rows. Thus, besides yielding results in both industry and commodity space, the model in (8) does not require the vectors $\alpha, \beta$ and $\mu$ in Statistics Canada's models with leakages.
    3. Computations were made on a PC with the Economists' TOOLKIT software by DIA Inc. and using 1988, medium level public use tables supplied by Statistics Canada in LOTUS 1-2-3 spreadsheets.
