

Research Note/Note de recherche

A Concise Description of Statistics Canada's Input-Output Models

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The Input-Output Division of Statistics Canada builds, maintains and uses two input-output models on a regular basis: the national model and the inter-provincial model. Clients of the Input-Output Division may conduct simulations with these input-output (I-O) models on a cost-recovery basis. The Division also maintain a cost-push inflation I-O model and a Net Price Index (NPI) I-O model that calculates taxes, subsidies and custom duties by categories of final demand, primarily for use by the National Accounts and Environment (NAE) Division.¹ Also, in the past, the Division has constructed an I-O embodied energy model. The NAE Division uses its own augmented I-O model to trace economic activity related to greenhouse gas emissions (Smith 1991). The Analytical Studies Branch uses its Commodity Tax (COMTAX) I-O model to provide parameters to its Social Policy Simulation Model for tax and transfer analysis (Bordt 1990).

These are all essentially open output determination models. A partially closed version was discontinued several years ago for reasons explained in Input-Output Division (1991), and Poole, Rioux and Simard (1994).

This text supersedes Hoffman et al. (1981) and Statistics Canada (1987). We cover the specification of the national model incorporating multiplicative

I thank René Durand and Terry Heaps for comments. Pierre Mercier's knowledge and clarity of thought made writing this text much easier. I claim all remaining errors and ambiguities.

1. See Durand (1988) and Durand and Rioux (1994).

leakages and unspecified leakages as introduced by Mercier, Durand and Diaz (1991). The interprovincial model is multi-regional and is explained in more detail in this paper than previously. First time users and clients of the Division seeking a less technical exposé and suggested uses of the I-O models may wish to consult Poole (1993). The purpose of this note is to further explain the intricacies of an I-O model based on rectangular commodity by industry I-O tables and to assist analysts and researchers conducting I-O simulations or building their own I-O models.

We begin by presenting the textbook Leontief model that carries most of the intuition necessary to understand the Canadian models constructed from rectangular tables. We then present the accounting identities and derived relationships that make up the national model and explain additional complexity of the interprovincial model. We show how to calculate industry output multipliers and how to derive producer prices from purchaser prices. In the conclusion, we discuss the limitations of the model and indicate an approach for model builders using public I-O data proposed by Damus (1993).

The Text Book Leontief Model

In a text book square Leontief input-output model, each industry (or sector) produces a single commodity and each commodity is produced by only one industry. One calculates the $m \times 1$ gross output vector g for m I-O industries that would satisfy the $m \times 1$ final demand vector f and the indirect industrial demand Ag in the following equilibrium equation for a closed economy:

$$g = Ag + f \quad (1)$$

where the $m \times m$ technical coefficient matrix A contains coefficients a_{ij} which represent the quantity of i th good produced by the i th industry needed to produce one unit of the j th good by the j th industry. The solution to equation (1) is:

$$g = (I_m - A)^{-1}f \quad (2)$$

where I_m is a $m \times m$ identity matrix. We refer to the matrix $(I_m - A)^{-1}$ as the multiplier or impact matrix, or traditionally, as the Leontief inverse. It shows the direct and indirect output requirements to meet the final demand f . In the impact matrix is the iterative process of an infinite number of successive demands for intermediate goods and services expressed by the power series expansion:

$$\begin{aligned} g &= f + Af + A(Af) + A^2f + \dots + A^kf + \dots \\ &= (I_m + A + A^2 + A^3 + \dots + A^k \dots)f \end{aligned} \quad (3)$$

As k tends to infinity, A^kf tends to zero. In the first round of production, f represents the direct output requirements to meet final demand f . In the second round of production, the vector Af represents the direct input requirements that furnish the goods and services necessary as inputs in the production of f , and so on until the term A^kf converges to zero.

The National I-O Model

The Canadian I-O model, by comparison to the text book model, is more complex. It is based on the rectangular structure of the I-O tables which have 627 commodities and primary inputs by 216 industries by 136 final demand categories at the most detailed level. Furthermore, we subtract leakages such as imports. Generally, we use the Canadian open I-O model to calculate the total output by business sector industry necessary to respond to a given final demand expenditure or industry output shock within a defined period; usually one year which corresponds to the time frame of the annual I-O tables, from which we estimate the parameters of the I-O model.

For expository purposes, we consider that the rectangular Canadian I-O model is composed of n non-primary commodities and m industries.²

Equilibrium Condition and Definition of Variables

We start with a real, static economy in equilibrium. Total supply of products equals total demand or disposition of goods and services in the following accounting identity:

$$q + m_D + m_R + a + v + s \equiv Bg + e + x_D + x_R \quad (4)$$

where the $n \times 1$ commodity vectors are defined as:

q	=	total domestically produced goods and services;
m_D	=	imported goods and services used domestically;
m_R	=	imported goods and services that are re-exported;

2. At the Worksheet (W) level, $n = 620$, of which six commodities are non-competing imports such as cotton, and there is one commodity of unallocated imports and exports. There are seven primary inputs and transfers (taxes and subsidies).

- a = government produced goods and services;
 v = value of withdrawals from inventories;
 s = unspecified leakages (incomes of non-profit organizations, production from the sale of used vehicles and aircraft parts, services sold to the public by educational institutions, etc.);
 Bg = intermediate demand of goods and services associated with the level of total domestic production by industry contained in the $m \times 1$ vector g and the technical coefficients in the commodity by industry ($n \times m$) matrix B ;
 e = domestic final demand; that is, the sum of personal expenditures, gross fixed capital formation, government spending, and additions to inventories, excluding unspecified leakages;
 x_D = domestically produced exports;
 x_R = re-exports.

In the final demand table, leakages are entered as negative values, although it may be more appropriate to think of them as an exogenous source of supply outside the business-sector. In addition to the unspecified leakages, there are sometimes other anomalies in the Canadian I-O tables. (For example, the value of imported gold exceeds the sum of domestic use and re-exports. This implies that some domestic exports of gold must come from imports, which, by definition, should not be the case). For some other goods, re-exported values exceed imported values. Mercier, Durand and Diaz (1991) deal with this problem by allocating imports that exceed domestic consumption and observed re-exports to re-exports. Re-exports that exceed imports are allocated to domestic exports.

To develop the model, we make several arbitrary assumptions about industrial production. We assume that production by industry is equal to the matrix of the domestic market share matrix multiplied by the vector of production by commodity:

$$g \equiv Dq \quad (5)$$

where D is an $m \times n$ domestic market share matrix. $D = V^o \hat{q}^{o^{-1}}$, where V^o is an $m \times n$ matrix of observed (historical) output.³ By construction, $\iota'_m D = \iota'_n$, where ι'_j is a transposed $j \times 1$ vector of ones, that is, the sum of the market shares of all industries for each commodity is equal to unity. In effect, we

3. In the past, the I-O Division has referred to the output table as the Make matrix, and the input table as the Use matrix. Some of us feel that this jargon confuses outsiders, so we prefer to use output tables to designate the Make matrix and input table to designate the Use matrix. The symbol $\hat{}$ designates a diagonal matrix where the off diagonal values are all zeros.

postulate constant market shares for new production.

We also make the industry technology assumption that the value of the inputs of each industry are a fixed proportion of the value of the total output of the same industry:

$$U \equiv B\hat{g} \quad (6)$$

where U is an $n \times m$ matrix of inputs (which does not include primary inputs), and B is an $n \times m$ industry technology matrix. The industry technology matrix is calculated using observed data:

$$B = U^o \hat{g}^{o^{-1}} \quad (7)$$

where U^o is an $n \times m$ matrix of observed inputs, and g^o is a vector of observed gross industry output. Note that primary inputs and non-competing imports are absent in the B matrix.

Output and Leakage Determination Reduced Form Equations

Before deriving the reduced form equation for endogenous g , we simplify equation (4) by omitting re-exports x_R and the equivalent value of imports m_R as they have no impact on domestic production; and we combine government goods and services a and withdrawals from inventories v into one column of leakages l in the following equilibrium condition equation:

$$q + m_D + l + s = Bg + e + x_D \quad (8)$$

where $l = a + v$.

We illustrate three identity equations that allow us to estimate three representative leakage coefficients (the actual model contains several more leakage coefficients, see Mercier, Durand and Diaz, 1991). We estimate import coefficients using observed data in the following identity equation:

$$m_D^o \equiv \hat{\mu}(Bg^o + e^o) \quad (9)$$

where $\hat{\mu}$ is the $n \times n$ diagonal matrix of import coefficients.

In a similar fashion, we estimate the other leakage coefficients using the following identity:

$$l^o \equiv \hat{\lambda}(Bg^o + e^o + x_D^o - m_D^o) \quad (10)$$

where $\hat{\lambda}$ is the $n \times n$ diagonal matrix of other leakage coefficients. Note that government services and inventories can supply domestic exports x_D .

We estimate unspecified leakage coefficients using observed data in the following identity:

$$s^o \equiv \hat{\gamma}(Bg^o + e^o + x_D^o - m_D^o) \quad (11)$$

where $\hat{\gamma}$ is the $n \times n$ diagonal matrix of unspecified leakage coefficients.

To derive the reduced form equation of the open I-O output determination model for gross output by industry, first we substitute equation (9) into equations (10) and (11), and substitute the result and equation (9) into the equilibrium condition equation (8), pre-multiply the terms by the matrix D , and gather the endogenous vector g on the left-hand side:

$$g = [I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1} D(I_n - \hat{\lambda} - \hat{\gamma})[(I_n - \hat{\mu})e + x_D] \quad (12)$$

where I_j is the identity matrix of dimension $j \times j$.

The $m \times m$ matrix $[I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1}$ is known as the inverse matrix and is conceptually similar to the $(I_n - A)^{-1}$ matrix given in the text book Leontief model example. If we post-multiply the inverse matrix by $D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})$, we have the impact matrix:

$$[I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1} D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu}) \quad (13)$$

that links the exogenous final demand expenditure shock e to the endogenous vector of gross output by industry g . An abridged version of equation (13) applies to domestic exports x_D .

The reduced form equation for gross output by commodity is:

$$q = [I_n - (I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})BD]^{-1} (I_n - \hat{\lambda} - \hat{\gamma})[(I_n - \hat{\mu})e + x_D] \quad (14)$$

whose derivation we leave as an exercise. Note that the $n \times n$ matrix BD is equivalent to the matrix of technical coefficients A in the square Leontief I-O model.

The vector of endogenous import leakages can be estimated by introducing the values of the endogenous vector g and values of the exogenous shock vector e , if appropriate, into equation (9). The vectors of other specified and unspecified endogenous leakages can be calculated in a similar fashion.

GDP Components and Employment

The industry GDP components matrix Y is calculated by multiplying the matrix of primary input coefficients H times the gross output by industry g : $Y = Hg$. Note that H is calculated using observed data: $H = Y^o \hat{g}^{o^{-1}}$. We calculate direct GDP by industry Y_d by multiplying the matrix H by the vector of industry production delivered to final demand g_d : $Y_d = Hg_d$, where $g_d = D(I_n - \hat{\lambda} - \hat{\gamma})[(I - \hat{\mu})e + x_D]$.

Coefficients representing job-output ratios for paid and other than paid jobs are multiplied by the appropriate vectors of output to estimate total and direct jobs.

Interprovincial Model

The Statistics Canada interprovincial I-O model is a multi-regional I-O model similar to the national model but consisting of 12 regional economies (10 provinces and two territories) and has an interprovincial trade or regional commodity share matrix for each commodity.

Definition of Variables

First, we introduce new variables and, where appropriate, redefine existing notation. The following 12×1 vectors are regional concatenations of commodity length vectors for each region.

q_p	=	total current production of goods and services by the domestic business sector, classified by commodity.
l_p	=	total leakages other than imports (government production, withdrawals from inventory and other leakages; for example, recycled materials such as scrap metals).
m_{Dp}	=	imports of foreign goods excluding commodities that are re-exported.
$B_p g_p$	=	total intermediate demand for goods and services.
e_p	=	total exogenous final demand, including personal expenditures, gross fixed capital formation, additions to inventories, government expenditures, but excluding exports.
x_{Dp}	=	domestic exports.

The subscript p indicates the concatenated vectors or matrices of 10 provinces and 2 territories. Total production classified by industry g_p is a 12×1 vector of regional concatenations of industry length vectors for each region.

The 12×12 inter-regional domestic market share matrix is a block diag-

onal matrix:

$$D_p = \begin{bmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & D_{12} \end{bmatrix} \quad (15)$$

where the sub-matrix D_i is the $m \times n$ domestic market share matrix for region i . The $12n \times 12m$ inter-regional technology matrix is similar:

$$B_p = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & B_{12} \end{bmatrix} \quad (16)$$

where the sub-matrix B_i is the $n \times m$ industry technology matrix for region i . The $12n \times 12n$ regional commodity share matrix is the full matrix:

$$R_p = \begin{bmatrix} R_{1,1} & R_{1,2} & \dots & R_{1,12} \\ R_{2,1} & R_{2,2} & & R_{2,12} \\ \vdots & & \ddots & \vdots \\ R_{12,1} & R_{12,2} & \dots & R_{12,12} \end{bmatrix} \quad (17)$$

where each $n \times n$ diagonal sub-matrix $R_{i,j}$ shows the shares of total supply of commodities consumed in region j which were produced in region i . This matrix is not to be confused with the $12n \times 12n$ regional import share matrix:

$$\mu_p = \begin{bmatrix} \hat{\mu}_1 & 0 & \dots & 0 \\ 0 & \hat{\mu}_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\mu}_{12} \end{bmatrix} \quad (18)$$

where $\hat{\mu}_i$ is an $n \times n$ diagonal matrix showing the shares of the total supply of commodities consumed in region i which were imported from a foreign country. It is important to note that regional supply shares and import shares for any one commodity must, by definition, sum to 1 as in the following identity:

$$\iota'_{12n} R_p + \iota'_{12n} \hat{\mu}_p \equiv \iota'_{12n} \quad (19)$$

where ι_{12n} is a $12n \times 1$ vector of ones.

Equilibrium Condition and Basic Identities

At the national level, total supply (domestic production plus imports plus other leakages) equals total demand (intermediate demand plus domestic final demand plus domestic exports) in the following accounting identity:

$$N(q_p + m_p + l_p) \equiv N(B_p g_p + e_p + x_{Dp}) \quad (20)$$

where $N = (I_n, I_n, \dots, I_n)$ is a $n \times 12n$ matrix of 12 horizontally concatenated $n \times n$ identity matrices. This equilibrium condition follows from the fact, mentioned earlier, that regional supply shares and import shares for any commodity must, by definition, add up to one (see equation (19)).

We can rewrite equation (20) by pre-multiplying intermediate and final demand (not including domestic imports) by the regional supply matrix and deleting imports from the left-hand side. Total domestic supply by commodity is the sum of intermediate and final demand pre-multiplied by the regional supply matrix plus domestic exports:

$$q_p + l_p \equiv R_p(B_p g_p + e_p) + x_{Dp} \quad (21)$$

We exclude re-exports which are met directly by imports.

In addition to equations (19) and (21), the model is defined by three additional identities based on assumptions similar to the ones that underpin the national model:

- The regional domestic market share matrix relates output by industry to output by commodity:

$$g_p \equiv D_p q_p \quad (22)$$

- Domestic imports m_{Dp} are the product of the regional import shares matrix and regional total demand net of domestic exports. Observed values are used to estimate the import coefficients:

$$m_{Dp}^o \equiv \hat{\mu}_p(B_p g_p^o + e_p^o) \quad (23)$$

and do not include imports allocated to re-exports in their calculation.

- Other leakages l_p (domestic supply originating from government

production, inventory withdrawals, as well as unspecified leakages such as the sale of scrap metals, the sale of used cars, etc.) are the product of the regional other leakage share matrix $\hat{\lambda}_p$ and regional total domestic supply:

$$l_p^o \equiv \hat{\lambda}_p [R_p(B_p g_p^o + e_p^o) + x_{Dp}^o] \quad (24)$$

Final Demand Deliveries and Direct Output

Final demand interregional trade by commodity is contained in the $12n \times 12n$ matrix:

$$P_d = R_p \hat{e}_p \quad (25)$$

The subscript d refers to direct effects; that is, production for delivery to final demand. The total supply of commodities by region to final demand is captured in the $12n \times 1$ vector:

$$s_d = R_p e_p + x_{Dp} \quad (26)$$

To calculate the $12n \times 1$ vector of direct commodity output by the domestic business sector q_{pd} , we must remove that part of production met by government services, inventories, etc., so:

$$\begin{aligned} q_{pd} &= (I_{12n} - \hat{\lambda}_p) s_d \\ &= (I_{12n} - \hat{\lambda}_p) (R_p e_p + x_{Dp}) \end{aligned} \quad (27)$$

We calculate the vector of direct industry output or current business sector production deliveries to final demand by pre-multiplying the commodity vector by the regional domestic market share matrix $g_{pd} = D_p q_{pd}$.

The matrix of direct GDP components Y_{pd} is calculated by multiplying the regional block diagonal matrix of primary input coefficients H_p by the vector of industry production delivered to final demand g_{pd} . The vector of direct foreign imports is captured in the $12n \times 1$ vector:

$$m_{pd} = \hat{\mu}_p e_p \quad (28)$$

Output and Import Determination Reduced Form Equations

We pre-multiply both sides of equation (21) by the D_p matrix, subtract other leakages from both sides, and then substitute equation (24). Dropping the p subscript, we have:

$$g = D[R(Bg + e) + x_D - \hat{\lambda}[R(Bg + e) + x_D]] \quad (29)$$

After some manipulation, we arrive at the reduced form equation that solves the model for the $12m \times 1$ vector of gross output in response to an exogenous shock from the $12n \times 1$ vectors of domestic final demand e and domestic exports x_D :

$$g = [I_{12m} - D(I_{12n} - \hat{\lambda})RB]^{-1} D(I_{12n} - \hat{\lambda})(Re + x_D) \quad (30)$$

Although the import coefficients are explicitly absent from the gross output determination equation, they are implicitly present in the magnitude of the regional production shares matrix where the sum of its elements in the presence of international imports are less than one.

The reduced form equation for gross output by commodity is:

$$q = [I_{12n} - (I_{12n} - \hat{\lambda})RBD]^{-1} (I_{12n} - \hat{\lambda})(Re + x_D) \quad (31)$$

whose derivation we leave as an exercise.

Total endogenous imports by commodity are calculated by inserting the endogenous values of g and exogenous e into equation (23). Total endogenous other leakages by commodity are calculated by inserting the endogenous values of g and/or exogenous e and/or x_D into equation (24). To calculate these and other variables by industry classification, we pre-multiply by the D matrix.

GDP components and jobs are calculated in a similar manner to the national I-O model.

Multipliers

National Model

This section describes how national I-O model industry output multipliers are calculated, and builds on the previous exposition of the national I-O model. We repeat the reduced form equation (12) for gross output by industry:

$$g = [I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1} D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})e + x_D \quad (32)$$

and define direct industry output as:

$$g_d = D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})e + x_D \quad (33)$$

where g_d is a $m \times 1$ vector also referred to as business sector deliveries to final demand. By definition, an economic multiplier measures the impact on an endogenous variable from a unit shock (or change) on an exogenous variable. We define a unit vector of exogenous industry output, and diagonalise the vector so as to decompose the impact by industry of supply. Each industry produces \$1.00 of output, so $\hat{g}_d = I_m$. We then calculate a square $m \times m$ matrix of gross production by industry:

$$\begin{aligned} G &= [I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1} \hat{g}_d \\ &= [I_m - D(I_n - \hat{\lambda} - \hat{\gamma})(I_n - \hat{\mu})B]^{-1} I_m \end{aligned} \quad (34)$$

Every row of G is a measure of the impact on gross output by industry resulting from the business sector delivery to final demand of \$1.00 by the industry associated with that column. The matrix G is the same as the inverse matrix we presented earlier, is also called the industry technology inverse and is similar to the total requirements table published by the US Department of Commerce.

Generally, the diagonal elements of the inverse are greater than one, because industries supply some of the intermediate demand associated with the production of \$1.00 of goods delivered to final demand. In other words, industries provide some of their own input requirements somewhere in the upstream production chain. The impact on intermediate output of industries represented by off diagonal elements are all greater or equal to zero and less than one.

We calculate a row vector of output multipliers by summing the columns of matrix G , that is, $\iota'_m G = \tilde{G}$, where ι'_m is the transposed vector of ones of dimension $m \times 1$. The other multipliers are calculated as a transposed vector of coefficients times the G matrix:

$$h'_i G \quad (35)$$

where h_i represents the direct GDP or labour income multiplier, subsidy rate or indirect tax rate, or the job-output ratio that allows us to calculate the employment multiplier. In the case of GDP, h_i is the value of GDP at factor cost divided by total industry output, which is equal by definition to total industry input. The other multipliers are calculated in a similar fashion. The employment multiplier L is calculated using the observed job-output ratio $j_o./g_o.$; the I-O Division uses a denominator of \$1,000,000.00 of shock to minimise the number of decimal places. The dot backslash operator $./$ performs element by element division. Succinctly:

$$L = (j_o./g_o)' G \quad (36)$$

In a similar manner, the multiplier of capital stock K is calculated as:

$$K = (k_o./g_o)' G \quad (37)$$

Using these basic building blocks, a series of other multipliers can be calculated. The capital income multiplier is calculated by subtracting the labour income multiplier from the GDP multiplier. The net indirect tax rate is calculated by subtracting the subsidy multiplier from the indirect tax multiplier. The multi-industrial average wage rate is calculated by dividing the labour income multiplier by the employment multiplier.

The Input-Output Division calculates the industry output multipliers and ratios at the "Worksheet" or W-level which contains 209 business sector industries. We calculate the multipliers at the more aggregate levels (link, medium and small) by weighting the W-level multipliers by the value of gross production of each industry within the aggregate.

Interprovincial Model

The provincial model I-O industry output multipliers are calculated in a similar fashion to the national I-O multipliers. We start by repeating equation (30):

$$g = [I_{12m} - D(I_{12n} - \hat{\lambda})RB]^{-1} D(I_{12n} - \hat{\lambda})(Re + X_D) \quad (38)$$

We assume the model is shocked with one dollar of delivery to final demand, which we diagonalise in order to produce a $12m \times 12m$ gross output matrix of sub-matrices:

$$G = [I_{12m} - D(I_{12n} - \hat{\lambda})RB]^{-1} I_{12m} \quad (39)$$

where

$$G = \begin{bmatrix} G_{1,1} & G_{1,2} & \dots & G_{1,12} \\ G_{2,1} & G_{2,2} & & G_{2,12} \\ \vdots & & \ddots & \vdots \\ G_{12,1} & G_{12,2} & \dots & G_{12,12} \end{bmatrix} \quad (40)$$

and $G_{i,j}$ is the square industry by industry ($m \times m$) sub-matrix of production of industries in region i for delivery to industries in region j .

Gross output multipliers are calculated by summing the columns which results in the $1 \times 12m$ row vector $\iota'_{12m} G$. The other multipliers are calculated by

multiplying each element of G by the appropriate coefficient or ratio. We can obtain intra-regional multipliers for specific provinces and territories by summing the rows of the diagonal sub-matrices, that is, G_{ij} , where $i=j$.

Producer Prices, Margins and Purchaser Prices

Until now, we have ignored prices and margins to simplify the exposition. The I-O models function in producer prices, whereas most observed market transactions occur in purchaser prices. To calculate the value of commodities in producer prices, we must remove various margins including taxes and place them in separate commodities. The seven margins are: taxes, gas, transportation, storage, pipeline, wholesale, and retail. Suppose we wish to shock one of the models with an exogenous vector priced in purchaser prices. A matrix of margin rate coefficients calculates the margins and allocates them to the appropriate margin commodities.

Let f_u be an $nx1$ vector of exogenous commodity purchases at final demand valued in purchaser prices, and let f_o be the same $nx1$ vector of exogenous commodity purchases valued in producer prices. To calculate producer prices, we subtract taxes and then subtract other margins that we reallocate to the margin commodities:

$$f_o = [I_n - (\iota_n' \Gamma) - \hat{\varphi}] f_u + \Gamma f_u \quad (42)$$

where taxes $\tau_d^o = \hat{\varphi} f_u^o$ are calculated by the $nx1$ vector of commodity indirect tax rates, φ , applicable to domestic final demand expenditures, and Γ is a nxn diagonal matrix of margin rates, where non-margin rows are empty and margin commodity rows contain the appropriate margin rates applicable to domestic spending.

By construction, the following identity is observed:

$$\iota_n' f_u^o \equiv \iota_n' f_o^o + \iota_n' \tau_d^o \quad (44)$$

Fictive Commodities and Industries

The Canadian I-O tables contain fictive commodities produced by fictive industries. These activities represent real economic activities that are difficult or impossible to observe directly. No value-added activity is directly associated with the production of these fictive goods.

As of this writing, the national model does not contain fictive industries or

commodities. All fictive activities are translated into their real components before the parameters of the model are calculated. The model cannot be shocked with these commodities.

Conclusion

The I-O models maintained by Statistics Canada are static and linear with no economies of scale or technical progress. Relative prices are absent, and no scarcity constraints are placed *a priori* on resources. I-O model "shocks" can only reproduce the average structure of the economy and not effects at the margin because of the linearity of the model and absence of feed-back effects (as might be the case in a macroeconomic model).

The I-O models are accounting models, capable of tracing intersectoral and interindustry exchanges with much detail (up to 216 industries by 627 commodities, including primary inputs). A balancing run of the model should reproduce gross output by industry or commodity as observed in the I-O tables.

Outside users generally construct their own models with public I-O tables, where we have suppressed some cells of information due to confidentiality requirements. To respect the basic accounting identities, outside users may wish to follow the procedures described in Damus (1993) and construct a dummy or fictive confidential industry and a confidential commodity.

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