On the Optimal Locational Policy for the Offshore Firm Entering a Foreign Market Area

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Many international economies have experienced trade liberalisation and reductions in trade barriers, for instance through the creation of such bodies as the European Union (EU) and the North American Free Trade Agreement (NAFTA). In this development, market areas have been enlarged beyond each nation's boundary. It can be expected that many firms would change their locational strategies in response to these new market areas. For instance, one firm may judge that it will be able to sell goods to a new market as a result of the expansion of its market area, and decide to export the goods by increasing production in its existing facilities. Another firm may build a new branch factory within a new market area to carry out its production and sales activities. Yet another firm may lose the benefit of maintaining its branch factory in the new market area, because of tariff reductions, and switch its production from a branch factory back to the main factory by shutting down the branch factory.[1] Thus, trade liberalisation may affect firms' production and sales activities, and their locational strategies, in many ways.

In order to study the locational changes of firms' production and sales facilities, motivated by changes in market areas, the analysis in this paper is based upon the premise that market area analysis provides some useful insights. The market boundary approach is effective in considering such questions as: If a firm builds a sales facility for the goods destined for a remote market,[2] where will the optimal location of its new sales facility be within the market area? Secondly, how would an existing firm respond to the new firm's entry into the market area? Our objective is to analyse specifically the consequences of price variation and negotiation.

In the literature on market boundary shape analysis, Launhardt (1885) pioneered the systematic study of the shapes of market boundaries. Launhardt showed that the shape of a market boundary is primarily determined by the firm's location, the price of goods and the freight rate. Many studies in the twentieth century generalised from Launhardt's work to generate more accurate and specific shapes for market boundaries. Parr's (1995) recent study focused on the detailed combination of firm's location, price of goods and freight rate, including exceptional point and linear markets. Furthermore, Rauch (1991) and Krugman (1996) have dealt with the problems of the effects of tariffs and trade barriers on the location of firms and cities.[3]

The literature cited above indicates the usefulness of the analysis of market boundaries and market areas when considering firm location and central place locations . But these studies emphasize a firm's location, and do not inform us much about the economic implications. This is due to the fact that it is difficult to obtain the necessary information (i.e., sales and profit) necessary for economic analysis in 'complex-shaped' market areas.

We suggest that this problem can be alleviated to a certain extent by incorporating numerical calculation analysis. In market area analysis, Gambini et al (1967) used numerical calculation in the theoretical analysis of market area shapes. Dacey (1966) considered a "central place system" through one of the numerical computation approaches, viz., the Monte Carlo method, and greatly advanced central place theory. Numerical computation is, therefore, introduced in this paper in addition to mathematical treatment, thereby enabling us to derive the volume of goods sold and firms' profits in complex-shaped market areas. The analytical methods utilised since Launhardt are analytically compact, but they are hard to grasp visually. In order to render market boundary analysis more visual, rectangular cone surfaces are used in this paper to display variations in the prices of goods and the relationship between firms' location and market areas. In a nutshell, we attempt to enlarge the applicability of market boundary analysis by incorporating economic implications through Monte Carlo techniques.

In the following section, the basic assumptions for our analytical framework are presented. Then, under the scenario of exporting goods by a new firm that

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2.   Henceforth referred to as "the new firm".
3.   The following studies can be added to the list: Benson and Hertigan (1983), Hatzipanayoutou and Heffley (1991), Porter (1984) and Heffley et al (1993). Analysess of market area shapes include Greenhut and Ohta (1978), Ishikawa and Toda (1990), and Schöler (1993).

contemplates entering the market area, the optimal location of the new firm's sales facility is studied. In addition, the optimal location for a branch production factory when the new firm decides to produce and sell its goods in the new market area is discussed. Finally, we discuss possible measures which may be taken by existing firms in the market area against the newly entering firms. Pricing behaviour by the new firm is analysed when the new firm establishes a sales facility. Furthermore, how the existing firm negotiates against the new firm when the new entrant establishes a branch factory is considered.

Basic Assumptions and the Model

We first consider the locations of a new firm's sales facility and branch factory, and the existing firm's counter-measures against the new firm, under standard basic assumptions.

A firm is an organisation. The facilities of firms such as production plant or sales offices are establishments. We assume that the "existing firm" B is a single plant enterprise in a single location. Also, we assume that new firm A does not sell in any other markets, foreign or domestic.

The circle in Figure 1 shows the market area surrounded by the sea. Consumers in this country reside evenly with density 1, and purchase only one kind of good. Each consumer has the demand function (1) for this good:

$$q = a - p - tu \tag{1}$$

The quantity demanded is q, the consumer's maximum demand price is a, the price of the good is p, the freight rate is t, and the distance from the consumer to the selling firm is u.

The notation system in this paper is as follows: A and B for firms, lower case p for price, Q for quantity, t is the freight rate, k and F the marginal and fixed costs respectively, and subscripted italics designate locations such as *mill*, *market* or
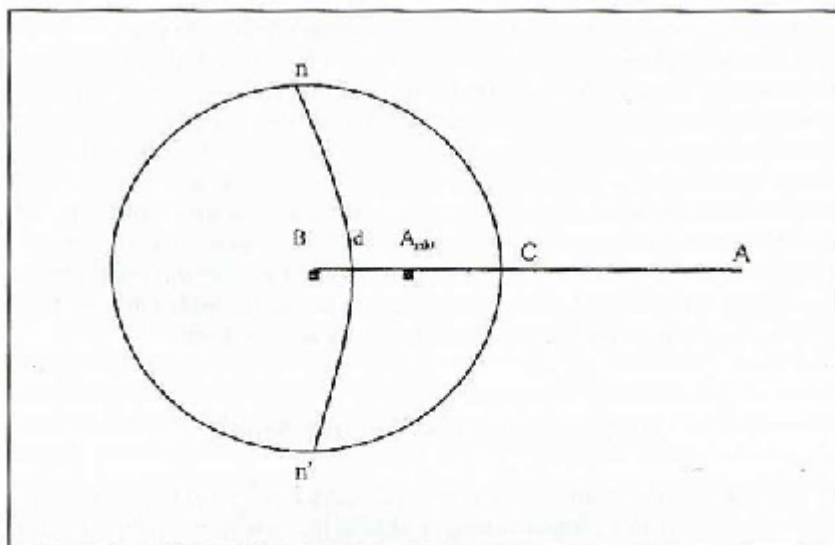
FIGURE 1 The Spatial Relationship Between Firms and Consumers
Note: Several features such as the arc from n to n' and d are referred to later in the text

port. For instance, the mill price of firm A becomes pA_{mill} and the price at its branch plant becomes pA_{branch}. The circular island/market area is country *SC*.

Within *SC*, there exists a firm B that monopolises this good, producing and selling it. Firm B, which is a single plant enterprise in a single location, is situated at the centre of the circle, B. It sells the good at the mill price pB_{mill}, and its cost function is:

$$c_B = kQ_B + F \qquad (2)$$

Here, c_B is the total cost, Q_B is the total quantity of the good sold, and k and F are marginal cost and fixed cost, respectively. Firm B's profit, Y_B, is given by:

$$Y_B = (pB_{mill} - k)Q_B - F \qquad (3)$$

The new firm A, which does not sell in any other markets, foreign or domestic, plans to enter country *SC* and sell the good there. It is located at point *A* in Figure 1. When the new firm A produces and transports the goods to country *SC*, it delivers them to port *C* of country *SC*.

When firm A's goods are sold to consumers, a change in the price levels is traced as follows. The goods depart from the factory at the price of pA_{mill}. The delivered price pA_{port} at port *C* is given by equation (4) since the transportation cost from firm A to port *C* is added:

$$pA_{port} = pA_{mill} + t_{sea}W \qquad (4)$$

where *w* is the distance form firm A to port *C*, and t_{sea} is the over water freight rate between firm A and port *C*.

The delivered price of the goods pA_{mkt} at the sales office A_{mkt} is given by equation (5) since the transportation cost from port *C* to the sales office A_{mkt}. is added:

$$pA_{mkt} = pA_{mill} + t_{sea}w + t_{land}h \qquad (5)$$

where *h* is the distance from port *C* to the sales office A_{mkt}, and t_{land} is the overland freight rate between port *C* and A_{mkt}. The freight rate differential arises from the difference in means of transportation and line-haul economies. It may not be too unreasonable to assume the freight rate differential to be $t_{sea} < t_{land} < t$ [4].

The new firm A's profit Y_A, when it sells the goods in country *SC*, is given by:

$$Y_A = (pA_{mkt} - k - t_{sea}w - t_{land}h)Q_A - F \qquad (6)$$

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4.   It is assumed that the scale of economy in transport from port *C* to sales facility A_{mkt} is larger than transport between the sales facility and the consumer. Therefore, freight rate t_{land} is assumed to be lower than *t*.

in which Q_A is the new firm A's total quantity sold.

On the other hand, when the new firm A directly establishes a branch production plant instead of exporting the goods to the island, the price from this branch factory has no direct relationship to the mill price at firm A, nor to the transportation cost between points A and C. In this case, the new firm A's profit becomes:

$$Y_{Abranch} = (pA_{branch} - k)Q_A - F \qquad (6a)$$

where pA_{branch} is the price at its branch plant.

Figure 2 shows changes in price levels and freight rates of the goods when firm A locates its sales office at point A_{mkt} and sells the goods in country SC. $p'A_{mkt}$ and $p'B_{mill}$ are referred to later in the text.

Optimal Location
of a New Firm's Sales Office and Branch Plant

Optimal Location of a New Firm's Sales Office

Let us now derive the profit-maximising location for the sales office A_{mkt}, when the new firm A exports the goods by establishing its sales office within country SC (Figure 1). In doing so, the relationship between firm A's profit and its sales office's location needs to be specified. In order to obtain this relationship, it is
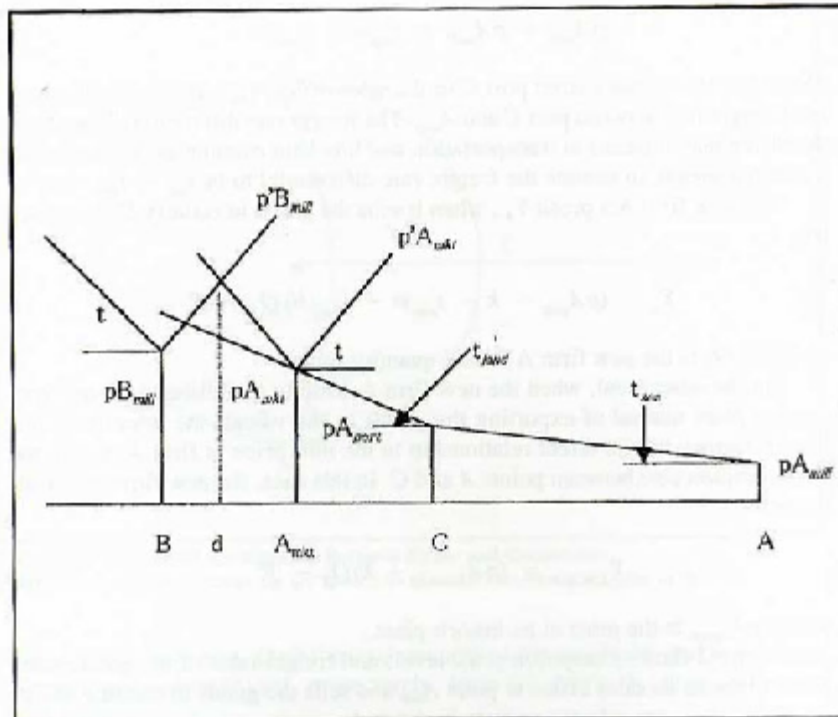

FIGURE 2 Firm Location and the Level of Prices
Notes: $p'A_{mkt}$ and $p'B_{mill}$ will be referred to later in the text

necessary to specify market area size and market boundary shape and to derive the quantity of goods sold. First, let us find the relation between office location A_{mkt} and market area size/boundary. Considering the market area shape and the existing domestic firm B's location, the range of the analysis can be confined to the line segment B-C in Figure 1.

When firm A tries to locate its sales office within country SC, the market area that A_{mkt} serves is determined (as demonstrated by Launhardt) by the relations between the price pA_{mkt}, the price pB_{mill}, their geographical distance L and the freight rate t. Figure 2 shows these relations.

It is assumed here that $pA_{mkt} < pB_{mill}$. Note that the price of the goods at its sales office, pA_{mkt} is determined by pA_{mill} and two different types of freight rates, t_{sea} and t_{land}. Consumers purchase from the firm that offers the lower price. Therefore, the boundary between firm B 's market area and firm A's sales office's market area is the set of points where the two delivered prices are equal. These price levels are depicted on the line segment B-C as $pA_{mkt} - p' A_{mkt}$ and $pB_{mill} - p'B_{mill}$ in Figure 2. Since the consumer's maximum reservation price is a (shown by the right-hand side demand curve in Figure 3), the whole range of prices on the plane are depicted by the two regular cones G-G'- pA_{mkt} and C-C'- pB_{mill} in Figure 3. Thus, the boundary for the two market areas can be obtained by the intersection of these two regular cones.

FIGURE 3 Price Represented by Regular Cone Surface
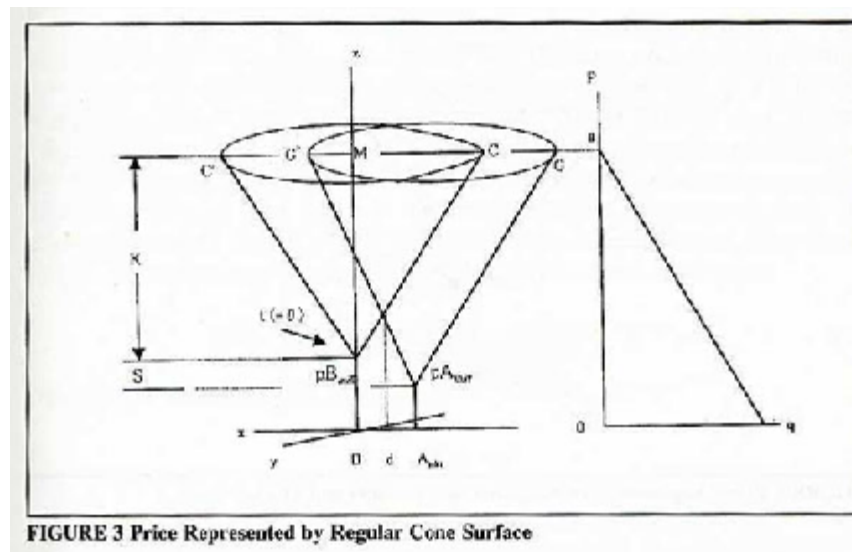
Miyasaka (1970) gave an elegant derivation of the boundary as the intersection of two regular cones in his seminal work. Miyasaka's book, published in Japanese, has not appeared in English. This work is introduced to English-reading regional scientists in Appendix A to provide for some logical development of the

following hyperbola equation (7).

When the new firm A tests alternative sales locations between port C and the point where the two competing prices are equal, the shape of firm A's market area boundary with firm B is a hyperbola for each possible location and is given by:

$$(L^2 - S^2\cot^2\theta)x^2 - S^2\cot^2\theta y^2 - L(L^2 - S^2\cot^2\theta)x +$$
$$\frac{1}{4}(L^2 - S^2\cot^2\theta)^2 = 0 \tag{7}$$

where L is the distance between firm A's sales facility and firm B, S is the price differential between firm A's facility and firm B, θ is the freight rate, and x and y represent the coordinate system with the centre of a circle, B, as the origin.
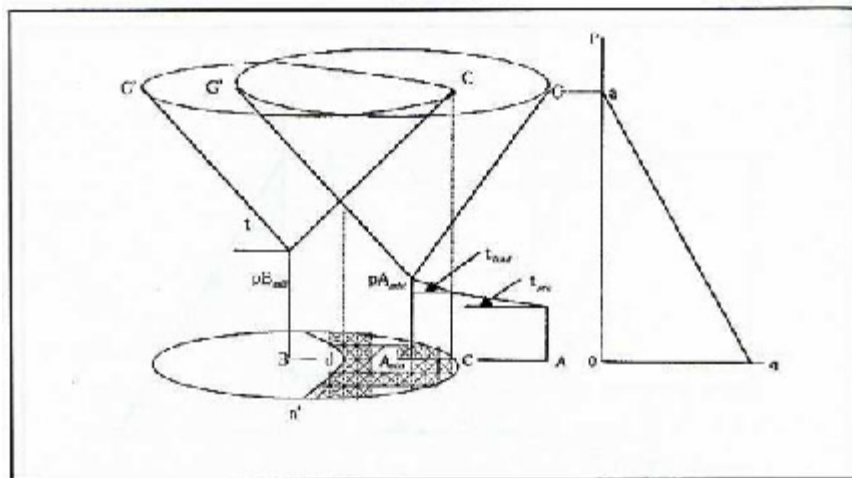

FIGURE 4 Prices Represented by Regular Cone Surfaces and Market Area

Under the assumptions of the prices and freight rates given in Figure 2, the shape of the market area boundary with firm B and sales office A_{mkt} is a hyperbola as given in equation (7). It is the n-d-n' curve in Figure 1. Since the hyperbola n-n' in Figure 1 is not sound, a carefully labeled plane view of the hyperbola would be useful at this stage (Figure 3). Furthermore, in order to avoid misunderstanding due to notational complexity, Figure 4 is presented. In Figure 4, Figure 1 is super-imposed over Figure 3 in gray tones.

As new firm A's sales office location moves from port C toward the centre of market area B, the hyperbola boundary shape will become a line since the price differential becomes smaller. This linear market area boundary is expressed as $L/2$ when firm A's sales office location reaches the point where the prices pA_{mkt} and pB_{mill} are equal.

The optimal location for new firm A's sales office must be the profit-maximis-ing point within the line segment B-C. It is cumbersome to derive this point be-

cause the computation of profits require s the total quan tity sold at each location to be known.

The total quantity of goods sold depends on the shape and size of the market area where consumers with the demand equation (1) reside. The shape and size of that area surrounded by the market area circumference and market boundary hyperbola are complex. Therefore, it is difficult to derive analytically the quantity sold in this market area. Thus, to derive the optimal location, we recommend the following procedure. First, each constant is given a specific value, and the numerical method is used to derive the total quantity of goods at each location. Then, new firm A's sales office profits at each location are calculated. Comparison of these profits values provides us with the profit-maximising location.

Each constant is given the following specific value, and profit at each location is derived by the Monte Carlo method [5]. Port C is situated at the coordinates (15, 0). The radius of the circular market area is 15. The new firm A's sales office price at port C, pA_{port}, is 2.5, and firm B's price pB_{mill} is 5. The freight rate t_{land} is 0.2, the freight rate t is 1, and therefore θ is 45°. The consumer maximum demand price a is 20, marginal and fixed costs are zero, and the transportation cost $t_{sea}w$ from (the production site of new) firm A to port C is zero per unit of goods. These variables are not important in this analysis because they will not affect the main logic of the analysis. Under these assumptions, the market area when firm A's sales office is located at the point (15, 0) is shown below. From equation (7) the market boundary hyperbola is given by:

$$25(x^2 + y^2) - (215.75 - 10x)^2 = 0 \qquad (7a)$$

The circular market area is defined by:

$$x^2 + y^2 = 15^2 \qquad (8)$$

Under the above specifications, the market area for the new firm A's sales

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5.   Our discussion turns here to the optimal location of the sales facility between C and B. Given unit density of the hypothetical population, this resolves into the calculation of the area of a hyperbola subtended by an arc. It is difficult to reach the answer by using integral calculus. In similar cases of difficult calculus, Gambini et al (1967) used numerical computation for the theoretical analysis of market area shapes. Dacey (1966) contributed to the enrichment of central place theory by applying numerical computation methods, viz., the Monte Carlo method, to the central place system. We argue that the Monte Carlo approach may be applied to compute sales and hence profits in the market area.

        For a reader who is unfamiliar with the Monte Carlo method of simulation, the ideas are as follows. The amount of sales is determined by the demand curve (equation (1)) and market area size. This amount is visually depicted by a complex-shaped volume on the market area. Computation of this value is difficult from a mathematical perspective. Since the Monte Carlo method is known to be effective in obtaining an approximate value of the volume of a complex-shape, it is applied in this paper. Incidentally, this application seems new, to the field of study of spatial market areas. Interested readers who want to see how Monte Carlo is used in this application can receive the program code through writing to the first author, Ishikawa.

office is that area surrounded by the curves (7a) and (8). Once the market area is established this way, the quantity sold and the profit of the sales office can be derived by the Monte Carlo method, based on the demand equation (1).

As the sales facility moves from port C towards the centre B, the price at the sales office and the market area's shape and size will change. These changes cause Firm A's profit to vary accordingly. Figure 5 depicts the relationship between the sales office's location and profit. From Figure 5, the following implications are derived. A firm that establishes its sales office and sells the goods should locate its sales office A_{mkt} at (10, 0), and the market area is the n-C-n'-d crescent shape in Figure 1. In this case, new firm A will obtain a maximum profit of 5,874. This new entry will benefit many consumers residing in the eastern half of the market area because they can purchase the goods at a lower price.
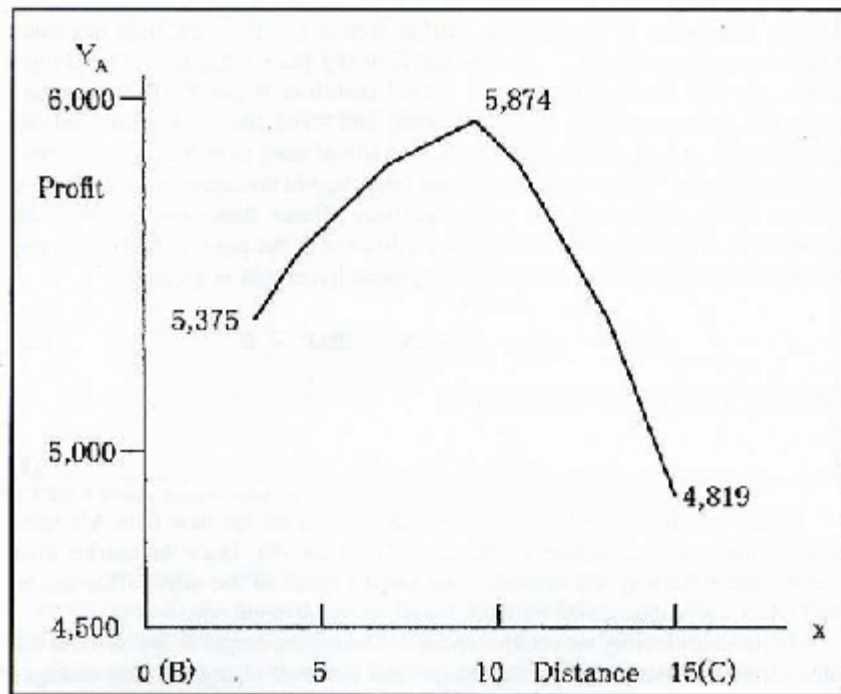

FIGURE 5 Sales Office Locational Changes and a Firm's Profit Changes

The Optimal Location of a Branch Factory

In this subsection, we derive the optimal location for a branch factory A_{branch}. This is the case where a new firm A establishes a branch factory that both produces and sells the goods in the market area in question.

Suppose that an existing firm B is located at the centre of the circle in Figure 1 with the price of the good at pB_{mill}. The new firm will build its production and

sales factory A_{branch} on the line segment connecting port C and the centre of circle, B. Let us consider that the conditions for production and sales may differ between a new firm and an existing firm. Suppose that these differences are reflected in the prices and freight rates of the goods, respectively. It is assumed that the freight rate is t_{firmB} when consumers visit the existing firm B, and that the freight rate is t_{firmA} when they visit the new firm's branch factory, with $t_{firmA} = t_{firmB}$. The difference in freight rates is made based on the assumption that the transport system around the existing firm is better developed than that of the new firm. For prices, it can be assumed that pA_{branch} is greater, less than or equal to pB_{mill}.
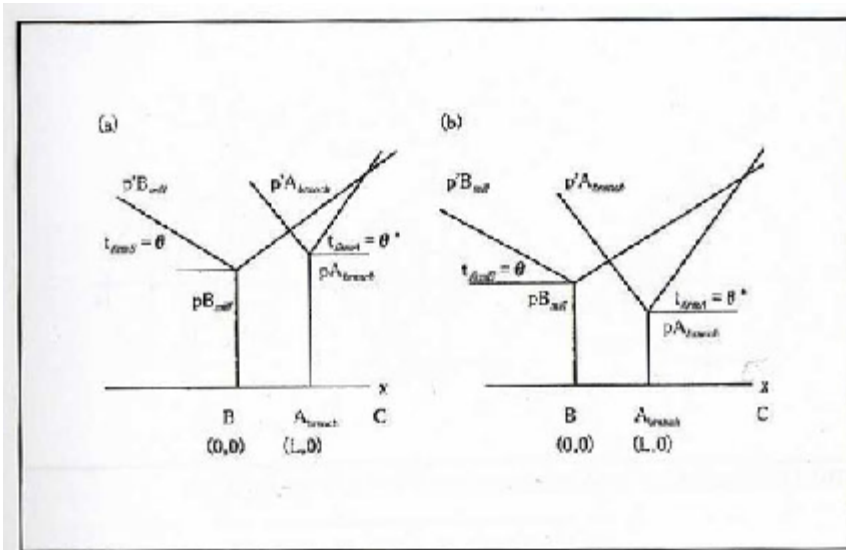

FIGURE 6 Firm Location and the Level of Delivered Prices

Analysis of the optimal location for the new firm's branch factory is made for the following three cases: (a) the new factory's price and freight rate are higher than the existing firm's (i.e., $pA_{branch} > pB_{mill}$, $t_{firmA} > t_{firmB}$: Case (a));(b) the new factory's price is lower than the existing firm's, but its freight rate is higher than the existing firm's (i.e., $pA_{branch} < pB_{mill}$, $t_{firmA} > t_{firmB}$: Case (b)); and (c) the new firm's branch factory price and freight rate are equal to those of the existing firm (i.e., $pA_{branch} = pB_{mill}$, $t_{firmA} = t_{firmB}$,Case (c)).

Case (a) The new firm's factory location and profit when its price and freight rate are higher than those of the existing firm

We analyse the new firm's factory location, and its sales and profit when its price and freight rate are higher than those of the existing firm. Figure 6 (a) shows the new firm A's price, pA_{branch}, and the existing firm B's price, pB_{mill}, together with their delivered prices ($pA_{branch} - p'A_{branch}$) and ($pB_{mill} - p'B_{mill}$). With such prices

and freight rates, the boundary shapes of the firms' market areas are represented by a family of horizontally extended ellipses. (See Appendix B for the derivation of boundary shapes of market areas.)
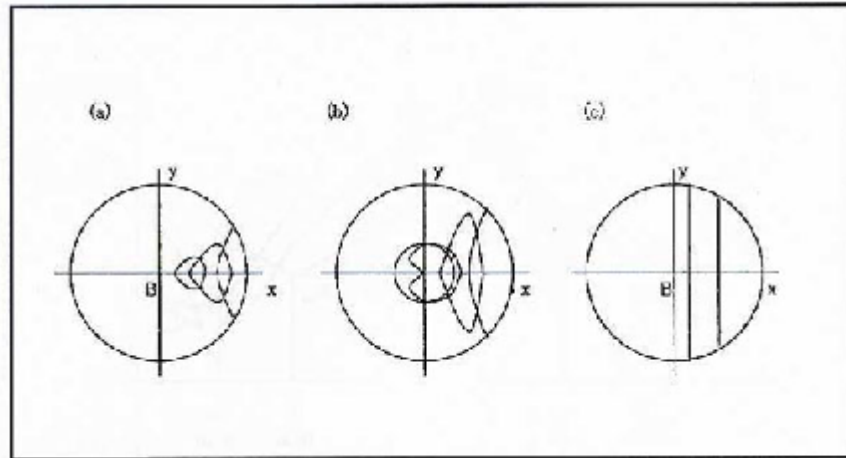

FIGURE 7 Firm's Location, Market Areas, and Market Boundary Shapes

The market area shapes formed corresponding to the new firm's factory location become complex, and the relationship between the factory and the market area sales and profits cannot be calculated by analytical methods. The new firm's optimal factory location can, however, be derived through numerical computation, as employed in the previous section. Each constant has the same value as before, except that the new firm's factory price, pA_{branch}, is 6 and the freight rate, t_{firmA}, is 1.5. In Figure 6(a), $(pA_{branch} - p'A_{branch})$ and $(pB_{mill} - p'B_{mill})$ show the levels of delivered prices of each firm.

Under these assumptions, the derivation of market area and boundary shape when new firm A's factory location moves from port C to the centre B yields the results depicted in Figure 7(a). As factory locations further from centre B are considered, the market area changes its shape and expands in size. The quantities demanded in these market areas are shown by the curve $q_7-q_8-q_9$ (Figure 8).

Once the shape and size of the market area are determined, the firm's profit at each location can be obtained. The relationship between the new firm's location and profit is described as the curve $r_7-r_8-r_9$ (Figure 9). This indicates that the new firm's factory will locate at the point (9, 0) as that brings the maximum profit of 5,286.

Case (b) The new firm's factory location, sales and profit when its price is low and its freight rate is high
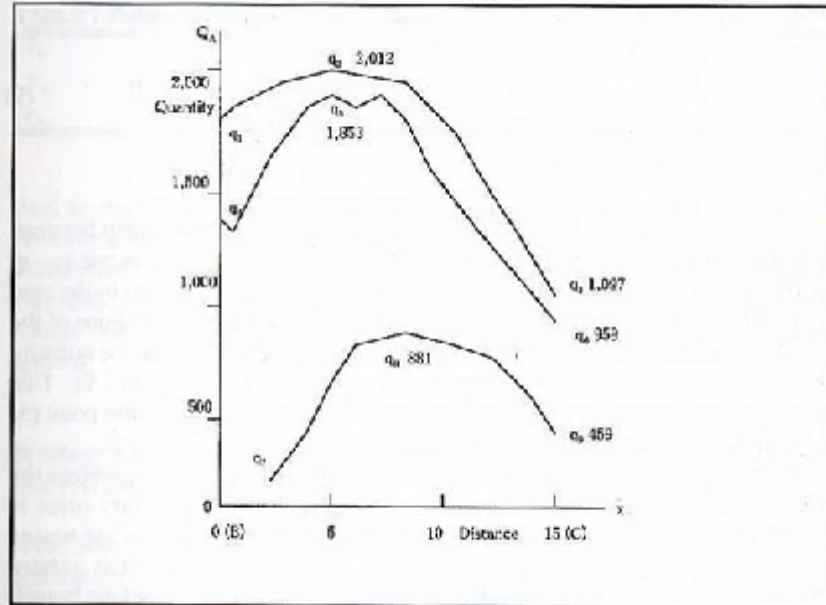
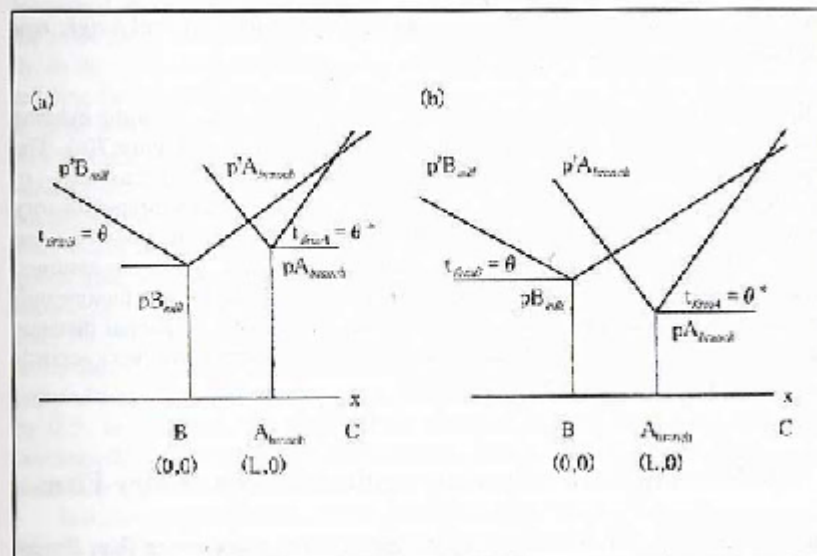**FIGURE 8 New Firm's Factory Location and Quantity of Goods Sold**

FIGURE 9 New Firm's Factory Location and Profits

Let us suppose that the entering firm's factory price is lower, and the freight rate is higher, than those of the existing firm, and that pA_{branch} is 2 and t_{firmA} is 1.5. In Figure 6(b), (pA_{branch} - $p'A_{branch}$) and (pB_{mill} - $p'B_{mill}$) show the levels of delivered prices of each firm. Changes in the shapes of market areas when the new firm's branch factory moves its location along the line *B-C* are depicted in Figure 7(b). As shown, the market boundary will mutate, changing shape from a circle to a limacon, to a vertically extended ellipse as the branch factory moves from the centre *B* towards port *C*. A limacon appears as a unilaterally indented ellipse. It indicates the shape of the market area boundary where the new firm's factory delivered price equals the existing firm's mill price at the existing firm's location. This gives equation (9) whose market area shape is a limacon [6]:

$$(x^2 + y^2 - 2\tan^2\theta * \frac{Lx}{V})^2 - (\frac{2S\tan\theta}{V})^2(x^2 + y^2) = 0 \qquad (9)$$

where $V = \tan^2\theta * - \tan^2\theta$.

In the same manner as in the previous subsection, the relationship between the branch factory location and quantity demanded is shown by the curve $q_4 - q_5 - q_6$ (Figure 8). At any location, quantities demanded are greater than in the case where the new firm's factory price and freight rate are higher than those of the existing firm. The new firm's branch factory profit, obtained through the quantity demanded at each location, is given by the curve $r_4 - r_5 - r_6$ (Figure 9). This indicates that the new firm will decide to locate its branch factory at the point (5, 0), with the maximum profit of 3,706.

Comparison of this result with that of the previous subsection reveals the following: in our numerical illustration, when the new firm's factory price is lower than that of the existing firm, the branch factory A_{branch} will locate nearer to centre *B*, with greater resultant quantities. The maximum profit is greater when the branch factory's price is higher than the existing firm's, and the branch factory will locate farther from the existing firm.

Case (c) The branch factory location and profit when its price and freight rate are equal to those of the existing firm

If the branch factory mill price and freight rate are equal to those of the existing firm, the market area boundary becomes a line, as shown in Figure 7(c). The branch factory location, quantity sold and profit are shown by the curve $q_1 - q_2 - q_3$ (Figure 8) and the curve $r_1 - r_2 - r_3$ (Figure 9). In this case, the branch factory locates at (5, 0) and obtains the maximum profit 10,059. Comparison of these results with the results from cases (a) and (b) shows that, given the assumed values, quantities and profit are always maximised when the branch **TABLE 1 The**

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6.   The limacon shape has not been referred to elsewhere in the study of market area boundaries.
       Appendix C gives the derivation of the limacon in relation to the standard form.

Existing Firm's Price Decrease and Changes in Profit and Quantity Sold

P_B	5	4.75	4.5	4.25	4	3.75
Y_B	11,420	11,571	11,595	11,146	11,268	11,396
Q_B	2,284	2,436	2,577	2,623	2,817	3,039

factory mill price and freight rate are equal. It is clear that the optimal locational distance between the new firm's branch factory and the existing firm B will vary according to the values of price and freight rate.

The Existing Firm's Strategy against the New Entry Firm

Faced with a new firm A's entry into the market area, the existing firm B may have a strategy for responding to this move. One such strategy is for the existing firm to change its location immediately upon the emergence of a new firm A. However, such a strategy appears unlikely, because the existing firm already has established production and sales networks. This narrows down the possible short-run strategies for existing firm B. Its options are: (A) to change its price and freight rate, or (B) to negotiate with the new firm. Here, we consider these two options.

Strategy (A): Changes in profits when the existing firm's levels of price and freight rates are altered

If the new firm A supplies the goods from its sales office A_{mkt} to the market area, and the existing firm B responds by changing price, the levels of prices brought about can be analysed as follows.

If a new firm A is in a situation as discussed in the previous discussion of the locational analysis of sales office A_{mkt} (i.e., where freight rate t is the same, and the price pA_{port} is 2.5 at port C), it will try to locate its sales office A_{mkt} at (10, 0). In this case, the price at the sales office pA_{mkt} is 3.5. Given these values, the existing firm B will alter its mill price in order to increase profits. A decrease in its price will expand its market area, and the amount purchased by each consumer will increase. Profits increase through this increase in quantity sold. On the other hand, a decrease in price means a reduction in revenue per unit of goods. This could cause profits to decline. A change in price varies the market area shape and size. Using the Monte Carlo method to obtain the existing firm B's quantity of goods sold, changes in profits are derived with a decrease in price. Table 1 shows the existing firm B's price decrease and changes in its profits and quantity sold. As price decreases, sales increase, and profit reaches a maximum of 11,595 when the price is 4.5. This means that if the existing firm B tries to counter the entry of a new firm A by a change-of-price strategy, it ought to decrease its price by 0.5. In this case, the shape of the market area remains a hyperbola, and becomes flattened as the price differential decreases. This implies that the market boundary ap-

proaches the new firm A's sales office.

It is also possible for the existing firm B to increase profits by decreasing the freight rate. For instance, the existing firm B may achieve a freight rate decrease by increasing consumers' shopping access by enlarging and refurbishing its parking space. Although investment in parking space is required, a decrease in freight rate increases the amount purchased, and can be expected to bring about an increase in profits because the increased sales revenue exceeds the cost of parking lot improvement. In this case, revenue per unit goods is considered unaffected. Therefore, a counter measure to decrease freight rate appears recommended to ensure an increase in profits, as long as the level of investment required to decrease the freight rate is relatively low.

Strategy (B): Negotiation with a new firm

Suppose new firm A builds a branch factory A_{branch} within the market area and plans to enter the market area. As discussed above, its branch factory location is determined by the conditions of price and freight rate. Anticipating the new firm building a branch factory A_{branch}, the existing firm B may wish to maintain its price and freight rate and may negotiate with new firm A in order to minimise its reduction in profit due to firm A's entry.

Considering the negotiation between these firms, the information that the existing firm B has to go on in negotiation are the new firm A's branch factory location, that branch factory's profit, and the existing firm B's profit. These are described as follows. The new firm A's factory location and that branch factory's level of profit are determined by the conditions of price and freight rate. Given the values for each price and each freight rate used above, the branch factory location and its profit are derived through the Monte Carlo method, as shown in Figure 9. In the same manner, it is possible to derive the relationship between branch factory location and the existing firm B's profit under each price and each freight rate. For the values of price and freight rate used in Case (a) (see above), the relation between branch factory location and existing firm B's profit is described by the curve $r'_5–r'_6–r'_7$ (Figure 10). The curves $r'_3–r'_4$ and $r'_1–r'_2$ in Figure 10 represent the relationships in Case (b) and Case (c) below.

The existing firm B may use negotiation as a strategy under the following conditions. Since the existing firm B has no power to regulate the new firm A's branch factory location, the new firm A should locate its branch factory in order to maximise profit. Of course, the new firm A may negotiate with the existing firm B and change its planned location of a branch factory A_{branch} if the existing firm B offers a profit-maximising proposal. The existing firm B will negotiate based on the proposed branch factory location, the new firm A's profit, and its own profit (Figures 9 and 10).

Next, we examine the type of negotiations that are probable under the prescribed conditions of price and freight rate discussed above.

Negotiation and branch factory location in Case (a)

As discussed in the previous section, the new firm's branch factory A_{branch} is located at the profit-maximising point (9, 0) and its price and freight rate are both higher than those of the existing firm B. As factory location moves away from this point, its profit decreases. In Table 2 (a), y_A shows how much profit declines as the location moves away from (9, 0). On the other hand, the existing firm B's profit may increase or decrease. The values y_B show how the existing firm B's profit would vary as factory location moves from point (9, 0). If movement of the factory location from point (9, 0) results in an increase in the existing firm B's profit which is greater than the decrease in the new firm A's profit, firm B may negotiate with firm A about the locational possibilities of firm A's proposed branch factory A_{branch}. It would do so because it can increase profit relative to the anticipated decrease in profit.
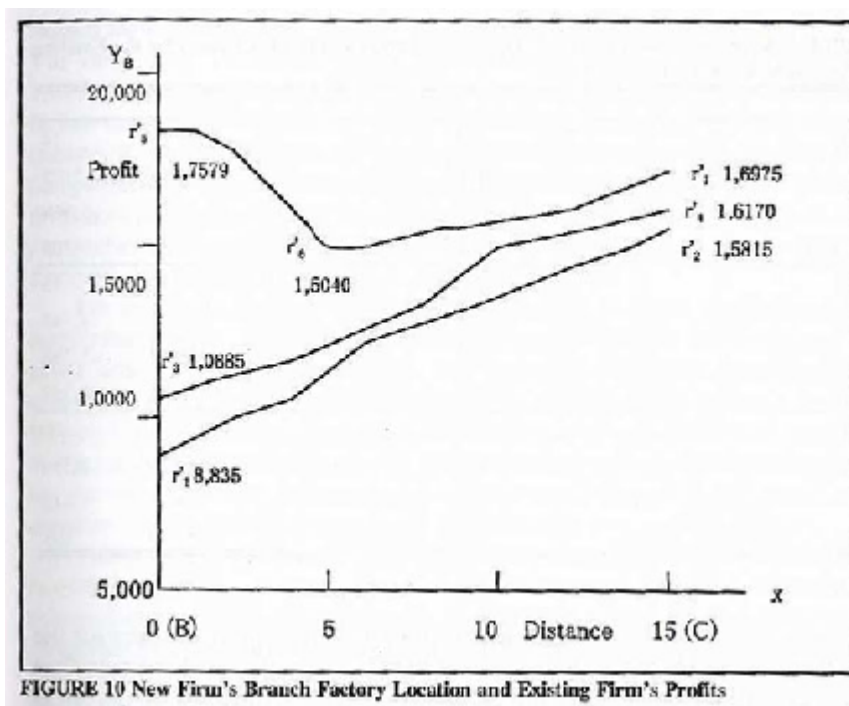

FIGURE 10 New Firm's Branch Factory Location and Existing Firm's Profits

When factory A_{branch} moves from point (9, 0), the results are shown as (y_B $- y_A$) in Table 2 (a). All of the values of ($y_B - y_A$) become negative. As a result, it is deduced that optimal branch factory location is at point (9, 0). The existing firm B is unable to negotiate with the new firm since the existing firm B's profit cannot compensate a decreased amount of branch factory profit upon the proposed factory's locational change.

TABLE 2 Departure from a Factory's Optimal Location and Profit Changes for the Existing Firm and New Firm's Branch Factory

	Distances							
(a) L	7	8	9	10	11	12	13	14
y_B	-450	-420	0	110	455	735	980	1185
y_A	-732	-234	0	-378	-606	-768	-1,338	-1,968
$y_B - y_A$	-1,182	-654	0	-268	-151	-33	-358	-783
(b) L	5	6	8	10	12	13	14	15
y_B	0	450	1,75	2,395	3,135	3,410	3,805	4,145
y_A	0	-78	-42	-368	-788	-1,136	-1,360	-1,788
$y_B - y_A$	0	372	1,.533	2,027	2,347	2,274	2,445	2,357
(c) L	3	4	5	6	7	8	9	10
y_B	-1,187	-605	0	820	910	1,540	2,035	2,350
y_A	-130	-80	0	-50	-205	-405	-845	-1,415
$y_B - y_A$	-1,317	-685	0	770	705	1,135	1,190	935

Negotiation and branch factory location in Case (b)

When the new firm's branch factory price is lower but the freight rate is higher than that of the existing firm B, the branch factory A_{branch} obtains its maximum profit at the location (5, 0). See the previous section for details. In Table 2 (b), y_B and y_A show a decreased amount of factory profit and profit changes for the existing firm B. This enables the existing firm B to negotiate with the new firm A about branch factory location if an increase in the existing firm B's profit is greater than the decreased amount of profit due to the branch factory locational change. In Table 2 (b), ($y_B - y_A$) shows the calculated results. As can be seen from Table 2, the existing firm B can increase its profit by 3,805 when branch factory location is changed from point (5, 0) to point (14, 0). On the other hand, the new firm's branch factory A_{branch} would decrease profit by 1,360. The net result, therefore, is that the existing firm B can still increase profit by 2,445. Using this net increase in profit as a leverage, the existing firm B can enter negotiations with the new firm A. When negotiating over branch factory location, any success in the negotiations will vary the market area shape and size for both the existing firm B and the new firm A's branch factory A_{branch}.

Negotiation and branch factory location in Case (c)

As discussed in the previous section, the new firm A's branch factory A_{branch} gets its maximum profit at the location $(5, 0)$ when the branch factory price and freight rate are the same as those of the existing firm B. Factory profit decreases as its location moves away from this optimal point. This decreased amount of profit and the existing firm B's profit changes due to the new firm's factory location moves are listed as y_A and y_B in Table 2(c). The existing firm B can enter into negotiations with the new firm A concerning firm A's branch factory location if an increase in firm B's profit exceeds a decrease in profit due to the branch factory's locational change. The calculated results are shown as $(y_B - y_A)$ in Table 2(c). This shows us that the existing firm B can increase profit by 2,035, if the branch factory location is moved from point $(5, 0)$ to point $(9, 0)$. In this case, the branch factory A_{branch} decreases its profit by 845. As a consequence, it is still possible for the existing firm B to increase profit by 1,190 after compensating for a decreased amount of branch factory profit. Based on this measured profit, the existing firm B enters negotiation with the new firm A. On completion of negotiations, the proposed branch factory A_{branch} may be moved from the originally planned point $(5, 0)$ to point $(9, 0)$.

For each case of the new firm's branch factory location, the findings are summarised below. Negotiation encourages the new firm A's branch factory to move further away towards the east. The quantity of the goods purchased increases in the eastern half of the market area where the new firm A's branch factory is built. When the location is altered through negotiations, the quantity of the goods purchased will increase, and the level of welfare in that area will go up. On the other hand, as long as the existing firm B does not lower price, the quantity purchased in the western half of the market area will not change.

Lastly, it is found that for each of the cases above locational change of the branch factory A_{branch} depends directly on profit changes for the new firm A's branch factory *and* the existing firm B. Furthermore, this profit change depends on market area size and shape, which in turn depends on the relationship between the mill price and the freight rate of the existing firm B and those of the new firm's branch factory A_{branch}. It appears that locational analysis based on market boundary shape is effective in examining a new firm's location and also the distance between firms.

Conclusion

When opportunities to market products in a new area arise, many firms may wish to enter the market area. Among the various forms of market entry, in this paper we examine two basic modes of entry: the sale of goods through locational shifts of a sales office, and also the production and sale of goods through building a branch factory. Locational problems under this mode of entry need to be discussed based on analysis of market areas. We need to call upon market boundary shape analysis to solve locational problems. This analytical method requires numerical

computation, and the Monte Carlo method is used in our locational analysis. Through this method, the sales function of a new firm and the location of its branch factory are analysed, and the optimal entry point is derived. Furthermore, it is found that the amount purchased by consumers increases and their level of welfare rises when a new firm enters the market area.

Our analysis has also examined price changes and negotiation as a counter-measure by the existing firm against the new firm. We discussed how prices should be changed and how a new entry location might be changed by negotiation. In a strategy of negotiation, price decreases somewhat when the existing firm tries to respond by changing prices, and the distance between the two firms increases. Shape analysis of market boundary is also used in this analysis.

In the above discussion of counter-measures, the existing firm's location is assumed to be fixed. However, it is anticipated that the existing firm would wish to move its location in the event of a new firm's entry if the entering firm is mobile. If such is the case, the mutually dependent relation between the existing firm and a new firm becomes complex, requiring the use of game theory and application of the research results.

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Appendix A

Shape Analysis of Market Area Boundaries

The consumer has a demand function (Figure 3) and purchases goods from the firm whose price is lower. Therefore, the market boundary is the point where the price of both an existing firm and the sales office built by a new firm are equal. The boundary line is a set of the intersections of the two regular cone surfaces representing the firms' prices (Figure 3).

Miyasaka (1970) provided an elegant mathematical treatment of solving two cone equations simultaneously. His method is applied to our problem here. In the following, the equations for the two regular cones (Figure 3) are first derived, and they are then used to confirm the market boundary shape. Assign the axes x, y and z to three dimensional space which shows the price levels of the goods in a circular market area of country SC. Fix the origin $(0, 0, 0)$ at the apex pB_{mill} of the regular cone surface that shows the level of the price for the goods sold by firm B. Since the maximum reservation price is a, and the existing firm B's mill price is pB_{mill}, the height of the regular cone surface is K ($=a - pB_{mill}$). The regular cone surface equation for firm B is given by :

$$K^2 (x^2 + y^2) = z^2 C^2 \qquad \text{(A1)}$$

where C is the radius of the regular cone surface base, which corresponds to port C in Figure 1. The slope of the regular cone surface line shows the freight rate. Since $K/C = tan\ \theta$, equation (A1) is transformed to:

$$(x^2 + y^2) \tan^2 \theta = z^2 \qquad \text{(A2)}$$

Similarly, the regular cone surface equation representing the price for sales office A_{mkt} that is located L away along the x axis from the origin $(0, 0, 0)$ is:

$$((x-L)^2 + y^2) \tan^2 \theta = (z+S)^2 \qquad \text{(A3)}$$

where S is the mill price differential between firm A's sales office and firm B.

Solving the simultaneous equations (A2) and (A3) of the regular cone surfaces with respect to x and y provides the specific shape of firms' market area boundaries. By developing the right-hand side of equation (A3), we obtain:

$$((x-L)^2 + y^2) \tan^2 \theta = z^2 + 2zS + S^2 \qquad \text{(A4)}$$

Substituting the left-hand side of equation (A2) into z of equation (A4):

$$((x-L)^2 + y^2 - (x^2 + y^2)) \tan^2 \theta - S^2 = 2zS \qquad (A5)$$

Squaring again in (A5) leads to:

$$(((x-L)^2 + y^2 - (x^2 + y^2)) \tan^2 \theta - S^2)^2 = 4 S^2 z^2 \qquad (A6)$$

Substituting again in the left-hand side of equation (A2) into z^2 of equation (A6), and dividing it by $(1/Cot^2 \theta)^2$, gives:

$$(L^2 - 2Lx - S^2 Cot^2 \theta)^2 = 4 S^2 (x^2 + y^2) Cot^2 \theta \qquad (A7)$$

Transforming equation (A7) gives the hyperbola equation:

$$(L^2 - S^2 Cot^2 \theta) x^2 - S^2 Cot^2 \theta \, y^2 - L(L^2 - S^2 Cot^2 \theta) x +$$
$$(1/4)(L^2 - S^2 Cot^2 \theta)^2 = 0 \qquad (A8)$$

Appendix B

Shape Analysis in the Case Where Market Area Boundary Belongs to the Ellipse Family

When the market boundaries belong to the ellipse family, analysis of specific shapes requires numerical computation. The regular cone surface equation for the new firm A's delivered price when both the price and freight rate of the branch factory A_{branch} are higher than those of the existing firm B is given in the following equation :

$$((x - L)^2 + y^2) \tan^2 \theta * = (z + S)^2 \qquad (B1)$$

where $\theta *$ is the branch factory freight rate. Solving the simultaneous equations (A2) and (B1) provides the market area boundary. Assuming that $S = 1$, $\theta = 45°$ and $\theta * = 56.31°$ and substituting the location abscissa on the line segment B-C into L, the market area boundary is obtained through solving the simultaneous equations, and the specific shape is determined by the following procedure. Substitution is made from 0 through 15 into x in appropriate intervals. For each value of x, a fourth degree equation of y is developed. Solving this fourth degree equation with respect to y, and plotting the values on the graph gives the market area boundary shape, as in Figure 7(a).

Appendix C

Limacon Transformation

It is hypothesized that a new firm's freight rate is more expensive than the existing firm's and that the price of the goods sold by a new firm is lower by S than that of the existing firm. Let us find the shape of the market boundary when the delivered price at point B in the plane market for a new firm's goods is just equal to the price for the existing firm's goods.

The equation (A2) in Appendix A gives the regular cone surface for the existing firm. The surface for a new firm is given by equation (B1). The market boundary is obtained by solving the equations (A2) and (B1) simultaneously with respect to x and y. The same procedure used in Appendix A provides us with the following:

$$(V(x^{-2} + y^2)^2 \tan^2 \theta^* Lx + \tan^2 \theta L^{2+} S^2)^2 =$$
$$4 S^2 \tan^2 \theta (x^{-2} + y^2) \qquad (C1)$$

where $V = \tan^2 \theta^* - \tan^2 \theta$. Since we suppose that a new firm's delivered price just equals the existing firm's price at point B, $\tan \theta^* = S/L$. The equation (C1) is thus transformed into:

$$(x^2 + y^2 - 2\tan^2 \theta^* Lx/V)^2 = (2S \tan \theta /V)^2 (x^2 + y^2) \qquad (C2)$$

This equation (C2) represents a limacon that has never previously been mentioned as a probable shape in the market boundary literature.