

A Comparison of Localized Regression Models in a Hedonic House Price Context

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It is the nature of spatial analysis to be concerned with local variations in a 'global' context. Analysts are compelled by the notion that objects nearer to each other are usually more similar and have greater influence on each other than objects farther apart – that is, that there is invariably some spatial autocorrelation in the data (Cliff and Ord 1981). 'Global' models, frequently based on ordinary least-squares (OLS) regression, assume that a single best equation can be found that characterizes the relationships between variables in a dataset pertaining to objects or locations in a particular geographic space. However, recent spatial multivariate regression models have emphasised that parameters identified in local models may not resemble the stationary parameters estimated in 'global' models. That is, they are often nonstationary (Brundson et al 1996; Fotheringham et al 2002; Páez 2003).

This paper, based on the methodology and results of Farber (2004), compares the results of the application of a number of spatial multivariate models to two 'global' models in a hedonic house price context – that is, the adequacy of two models with nonstationary parameters are compared with two with stationary parameters. Although level of 'adequacy', or fit, is often in the eye of the beholder, in this case the comparisons are made with a number of distinct measures: multiple R^2 (or pseudo- R^2),¹ and its counterpart the sum-of-squared errors (Páez et al 2002a

1. The pseudo- R^2 is equal to the squared correlation coefficient between the vectors of predicted and observed values of the dependent variable.

and *b*); and, the Moran Coefficient (MC), which in this study is used to measure the spatial autocorrelation in the pattern of residuals for each model (Goodchild 1986; Cliff and Ord 1973; Moran 1948).

The models are applied in a hedonic house price context because this type of modelling is commonly used for taxation purposes in which good model fits (i.e. high R^2 s) are *sine qua non*; and errors that do not contain a spatial bias are preferred because they imply equitable estimation between neighbourhoods. Hedonic house price models are based on the premise that the value of a property is a function of the perceived (or implicit) value of its characteristics. Traditional hedonic modelling entails the use of OLS regression to estimate the values of characteristics falling into three categories: structural, accessibility, and neighbourhood (Fik et al 2003). The model has the general form:

$$P_i = f(S_i, A_i, N_i) + \epsilon_i \quad (1)$$

where, P_i is the price of house i , S_i , A_i , and N_i are vectors of structural, accessibility and neighbourhood attributes and ϵ_i is an independent and normally distributed error term.

Data and Variables

Hedonic house price models in general, and spatial multivariate models in particular, require large data sets in which the information has a high degree of geographic spread (Waddell et al 1993). This is because a basic attribute of the information is its location, and this information has to have the appearance of being likely to represent the variability in the human, economic and physical environment of the study area (McMillen 1996). In this study, the data set consists of 19,007 (i.e. all) freehold housing sales taking place between July 2000 and June 2001 in the City of Toronto. Figure 1 shows the wide spatial distribution of the sales points overlaid with zones of residential land-use.

Financial and structural attributes (over 100 variables) were provided by the Municipal Property Assessment Corporation (MPAC), and locations have been geocoded according to Teranet's parcel centroids.² To this data set has been added a large number of GIS generated variables, related primarily to neighbourhood (from census data) and locational characteristics derived by the researchers, such as distance to a number of different destinations and features in the built environment (Andrey 2003).

2. For more information on Teranet and their Ontario Parcel Database please visit their web-page at <http://www.teranet.ca>

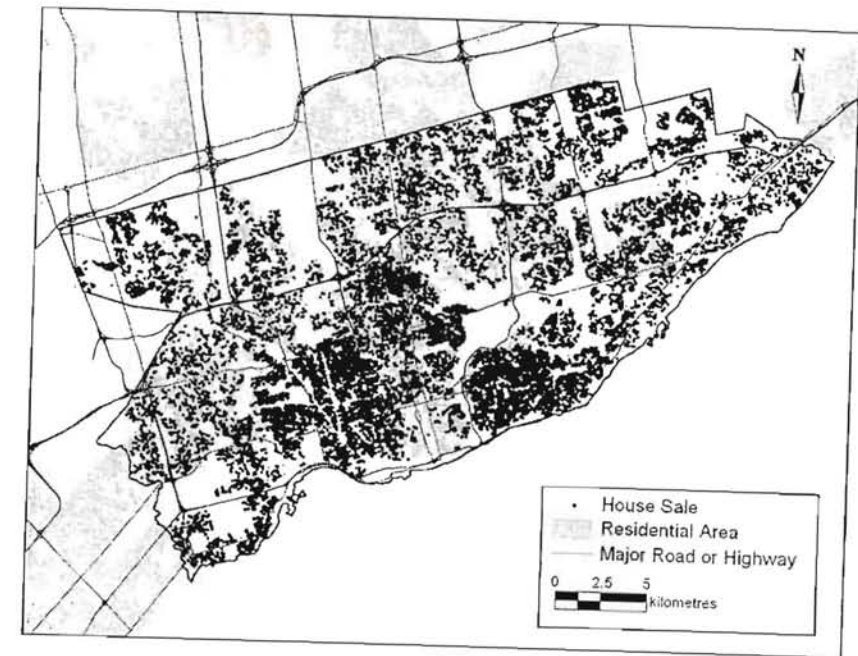


FIGURE 1 House Sale Location and Land Use

The dependent variable to be 'explained' is sales price -- generally improved capital values (ICVs) -- though there are a handful of sales of land without buildings where the prices reflect unimproved capital values (UCVs). Nevertheless, whether ICVs or UCVs, there is considerable variation in sales prices throughout the city (Figure 2) which in itself suggests that housing values in the city are related to both structural and locational/neighbourhood attributes.

'Global' Models

Standard Hedonic House Price Model

Echoing a process used by Des Rosiers et al (2000), many variable combinations were tested in search of an unbiased and stable standard 'global' hedonic house price model (hereby referred to as GOLS). The most powerful model, in which there is little multicollinearity at the 'global' level, consists of five housing/structural characteristics, two neighbourhood characteristics and two accessibility characteristics (Table 1).

Reading through Table 1, it can be observed that 83% of the variation in sales prices (normalized through a logarithmic transformation) is 'explained' by a model which includes: the area of the house (also normalized, the beta value indicating it to be the most strongly related to the variation in house prices in the context of

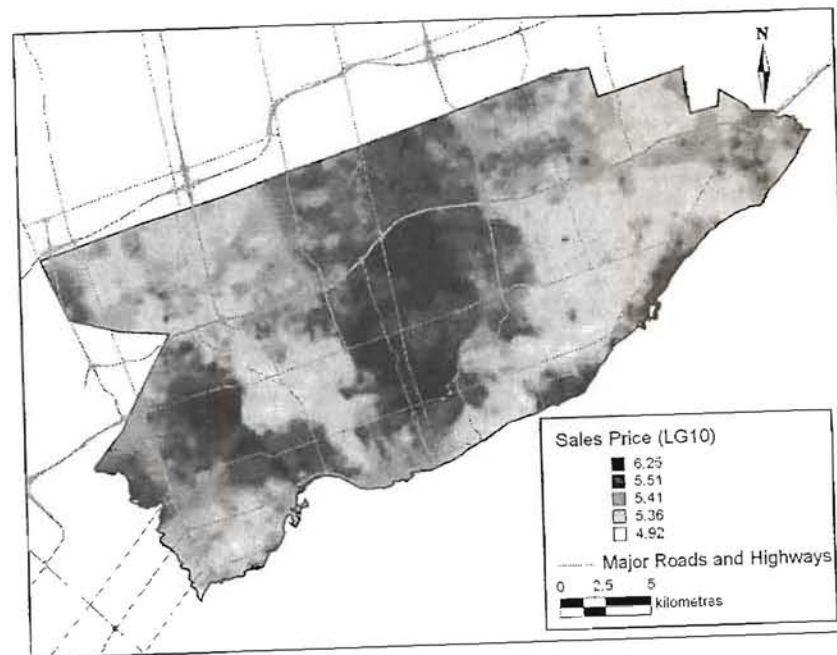


FIGURE 2 Spatial Distribution of Sales Prices in Cdn\$ (LG10)

TABLE 1 Parameters for the 'Global' Model

	Model	GOLS	
	R ²	0.831	
	SSE	109.9	
	B	Beta	T
CONSTANT	1.43E+00	--	63.07
Structural Variables			
LG_AREA	4.92E-01	0.428	106.32
AGEPROP	-5.37E-04	-0.078	-16.60
SALEDAT1	1.36E-03	0.028	9.36
LG_SIZE	1.96E-01	0.253	65.56
QUALITY	4.93E-02	0.138	33.94
Accessibility Variables			
DOWNDIS	-1.11E-05	-0.362	-72.09
DISMAL	-7.31E-06	-0.075	-22.54
Neighbourhood Variables			
LG_HHINC	3.59E-01	0.305	76.79
PC FOR	-1.21E-01	-0.109	-26.51

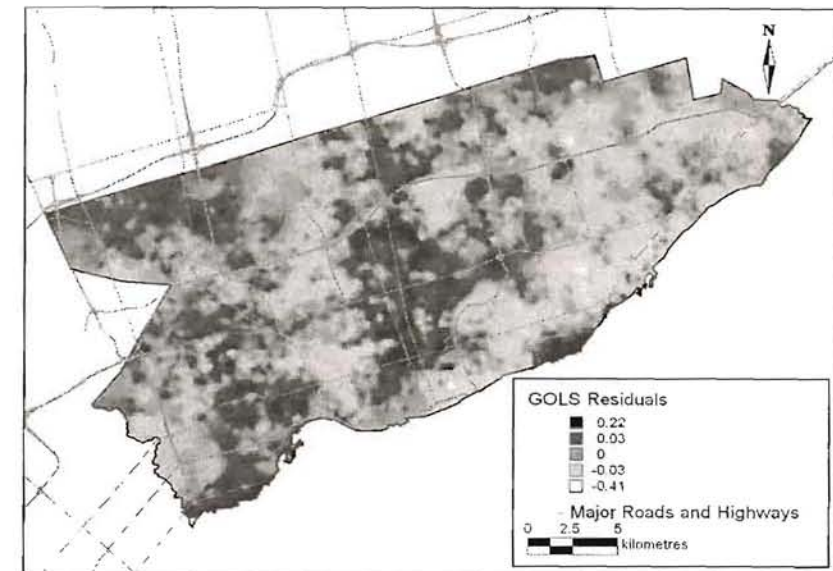


FIGURE 3 Surface of GOLS Residuals (MC = .240)

the model), the age of the property (in general, the older the property, the lower the value), the sale date in the 12-month period (a slight upward trend during the period), the size of the lot (also normalized, the larger the lot, the greater the value), an assessor's estimate of quality (quality has a positive association with house price), a composite measure of distance to the downtown (the sign and beta value suggesting that the city remains monocentric – the closer to the downtown, the higher the value of the property),³ distance to the nearest super-regional mall (the closer a regional mall the higher the value of the property, suggesting that some polycentrism is emerging),⁴ mean household income (also normalized, and indicating, not surprisingly, that high income neighbourhoods contain high value houses),⁵ and percent of foreign-born residents (included to capture Toronto's extraordinary multicultural nature, the 'global' parameter suggesting that, in general, DAs with high percentages of foreign-born residents are associated with lower value residential properties).

Although the multiple R^2 is quite high and the residuals (or errors) normally distributed, the residuals exhibit a high degree of spatial autocorrelation with a Moran Coefficient⁶ of 0.24 (Figure 3). This level of spatial autocorrelation suggests that one or more

3. Compare with Heikkila, et al (1989).

4. See Yeates (2000).

5. Neighbourhood variables based on Dissemination Areas (DA), which are the smallest geographic unit to which data are aggregated by Statistics Canada for the 2001 *Census of the Population* – they consist of one or more city blocks containing generally between 250-400 households.

6. Values of MC approach +1 when neighbouring observations are similar, -1 when they are dissimilar and approximately 0 when observations are randomly distributed over space (but not interpretable like a Pearson's R). In this study, near neighbours in the W_{ij} weights matrix are defined as those within 1000 metres of i .

significant spatial processes may have been left out, and/or their coefficients have been specified incorrectly during the modelling process. In the first case, the addition of more location based variables in the model could be associated with much of the residual spatial variation in sales price, such as a spatially autoregressive term representing local market values. In the second case, misspecification could be a result of nonstationary regression coefficients, and a switch to a local modelling framework may well provide a way of addressing this issue. Both avenues are addressed.

Global Model with a Spatial Autoregressive Regression Term (SAR)

In the previous section it was shown that the ability for an OLS model to satisfactorily predict residential property values could be reduced to the problem of specifying spatial variables that capture externalities associated with different locations in the urban fabric.⁷ For example, it may well be that the price of a house is a function of recent sales prices for similar houses in the neighbourhood, that i.e.:

$$P_i = \rho WP + \epsilon \quad (2)$$

where P contains an $n \times 1$ vector of spatially autocorrelated variables, W is a row normalized $n \times n$ connectivity matrix, ρ is the 'global' estimate of spatial interdependency, and ϵ is a normally distributed vector of error terms (Anselin 1988).

A more realistic function, termed a mixed autoregressive-regression model, is a hybrid between (2) and (1) which assumes that sales price is a function of both neighbouring values as well as its structural, neighbourhood and accessibility attributes:

$$P_i = \alpha + \rho WP + \sum \beta_S S + \sum \beta_A A + \sum \beta_N N + \epsilon \quad (3)$$

where the β terms are regression coefficients, α is a constant and all other terms are defined as above (Anselin 1988; Can and Megbolugbe 1997; Haider and Miller 1999; LeSage 2001). Unbiased estimation of the model is usually obtained by using a maximum likelihood procedure as outlined by Anselin (1988). However, OLS is often used in practice when using large datasets (Can and Megbolugbe 1997; Haider and Miller 1999). The spatial autoregressive methodology used in this report follows that of Can and Megbolugbe (1997: 214), who, in their analysis of a sample of 944 housing transactions in Miami MSA (FL), provide coefficient

7. Other ways of addressing this issue are through kriging and spatial error modelling, which basically hypothesize that the pattern of errors may in part relate to one or more unobserved (i.e. unknown) locational variables (Dubin 1992; LeSage 2001). The problem with these methods is that they are by their very nature obscure as to what this additional systematic pattern relates.

TABLE 2 Parameters of the SAR Model

	Model	SAR	
	R ²	0.878	
	SSE	79.4	
	B	Beta	T
CONSTANT	7.01E-01	--	33.31
LG_AREA	4.01E-01	0.348	98.24
AGEPROP	-4.96E-04	-0.072	-18.04
SALEDAT1	1.41E-03	0.029	11.42
LG_SIZE	1.36E-01	0.176	51.75
QUALITY	3.55E-02	0.099	28.52
DOWNDIS	-6.71E-06	-0.219	-47.87
DISMAL	-1.87E-06	-0.019	-6.60
LG_HHINC	9.93E-02	0.085	19.87
PC_FOR	-8.00E-02	-0.072	-20.44
LAG	4.49E-01	0.416	85.33

estimates based on OLS.

Since OLS can be used to estimate the above model, the only difference between (2) and (1) is the inclusion of a spatially-lagged price variable, WP . The creation of the lag variable is subjective, but for each house in the dataset it should adequately represent the distance-weighted average sales price of the surrounding units. In this case, the lag variable is calculated as the distance weighted average of the house's 10 nearest neighbours appearing in the dataset of 19,007 house sales.

The inclusion of the lag variable has several effects. There is an overall improvement in goodness-of-fit as indicated by an increase of R^2 to 0.878 and a reduction in the SSE to 79.4 (Table 2 compared with Table 1). While all of the coefficients in the model remain significant, they have lessened in importance with the addition of the spatially lagged variable which has a regression coefficient of 0.45, except for SALEDAT (which continues to indicate a 'global' upward trend in sale prices throughout the data collection period). The ability of the model to capture spatial processes has also improved as is indicated by a reduction in the MC for the residuals (Figure 4) from .240 in the GOLS model to .092 in the SAR model.

The SAR model is an improvement over the GOLS hedonic model.⁸ It results in an overall improvement to model accuracy, a reduction in spatial bias and probably a more realistic set of regression coefficients considering that neighbour-

8. A concern about the SAR model is that variables which correlate strongly with the dependent variable are likely also to correlate with the autoregressive term. However, for the purposes of comparing models in this paper, it was decided that a consistent set of variables must be retained from procedure to procedure.

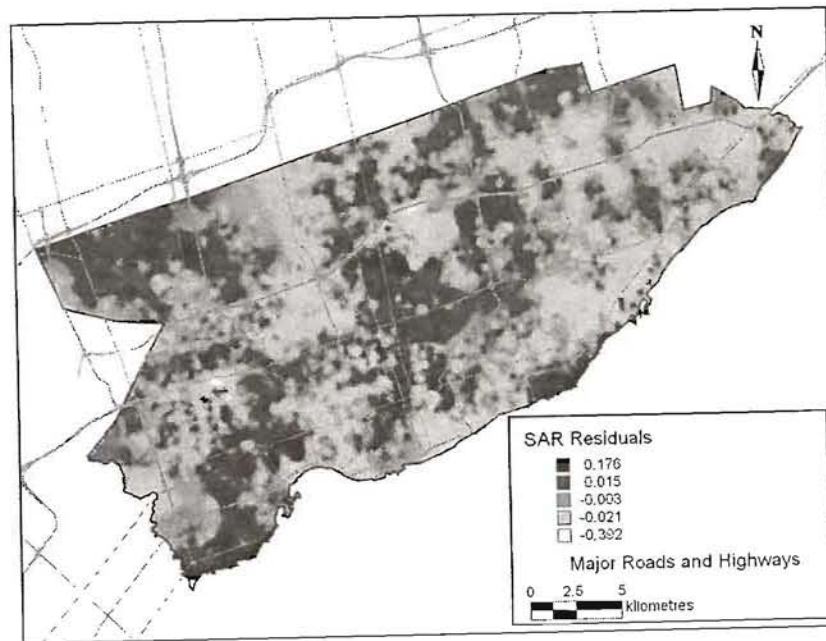


FIGURE 4 Surface of SAR Residuals (MC = .092)

hood impacts are made more explicit. Nevertheless, in spite of improvements to capturing neighbourhood variability, additional, and perhaps more interesting results might be achieved by moving from 'global' to local estimation (Pavlov 2000). In the following sections, the results of two local models of estimation are evaluated.

Local Models

Geographically Weighted Regression (GWR)

The first model of local regression used in this analysis is GWR, which consists of a series of locally linear regressions that utilize distance-weighted overlapping samples of the data (Fotheringham et al 1998). That is, for each observation, parameters are estimated such that the impacts of closer observations are stronger than the impacts from observations farther away – reflecting the central finding of the SAR model.

The GWR model is formally defined as:

$$P_i = \beta_{0i} + \sum_k \beta_{ki} X_{ki} + \epsilon_i \quad (4)$$

where P_i is the i th observation of the dependent variable, X_{ki} is the i th observation of the k th independent variable, ϵ_i is the i th value of a normally distributed error vector with mean equal to zero, β_{0i} is the constant estimated for local regression i , and β_{ki} is the regression coefficient estimated for regression i and variable k . This differs from ordinary least squares regression by utilizing distinct constants and regression parameters for each point, rather than 'global' parameters.

The estimation algorithm essentially iterates through n ordinary least square regressions, each one modified by a unique distance-decay weight matrix. Estimation thus takes the form:

$$B_i = (X^T W_i X)^{-1} X^T W_i P \quad (5)$$

where, β_i is the vector of estimated coefficients for observation i , P is the vector of observed dependent variables, X is the $n \times k$ matrix of explanatory variables, and W_i is a diagonal distance-decay weight matrix customized for i 's location relative to the surrounding observations.

This model has been evaluated utilizing software by, and described in, Fotheringham et al (2002)⁹: a bi-square weighting scheme was selected, and the same number of near neighbours (274) for each local regression was defined utilizing the software's cross-validation procedure. A traditional R^2 measure of goodness-of-fit cannot be calculated since the estimates are based on 19,007 local regressions, but there are several alternate methods.

The simplest two measures are the SSE, and a pseudo- R^2 . The SSE for the model is 52.7, the smallest of all the models tested in this paper. The pseudo- R^2 is defined as the squared correlation coefficient between the observed and the predicted values, and its value is 0.919 – it is, in effect, the cumulative effect of 19,007 regressions. The mean of the local R^2 's is 86%, with a maximum of 97% and a minimum of 51%. Diagnostics revealed that less than 3% of the local models had R^2 's lower than 70%, and only 30% of the models had R^2 's less than the R^2 for GOLS (83.1%). Furthermore, 30% of the models obtained R^2 's in excess of 90%, an improvement of 6 percentage points over the GOLS model.

At first glance, the pattern of local R^2 's (Figure 5) resembles that of the distribution of the dependent variable, LG_SALE (see Figure 2). But, as the overall goodness-of-fit is based only on the estimate for observation i in model i , the pattern of local R^2 's in Figure 5 does not indicate that the model's overall goodness-

9. As in Mennis and Jordan (2005).

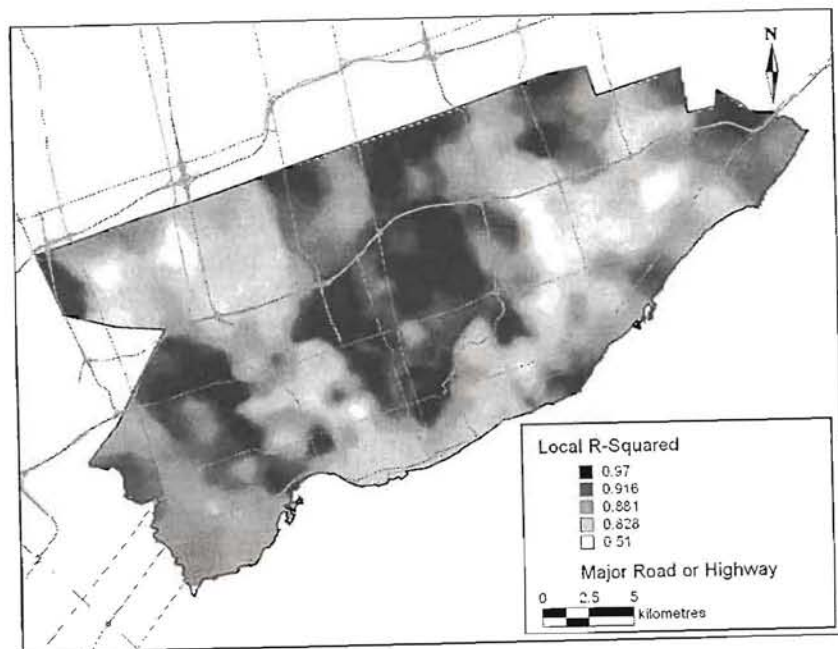
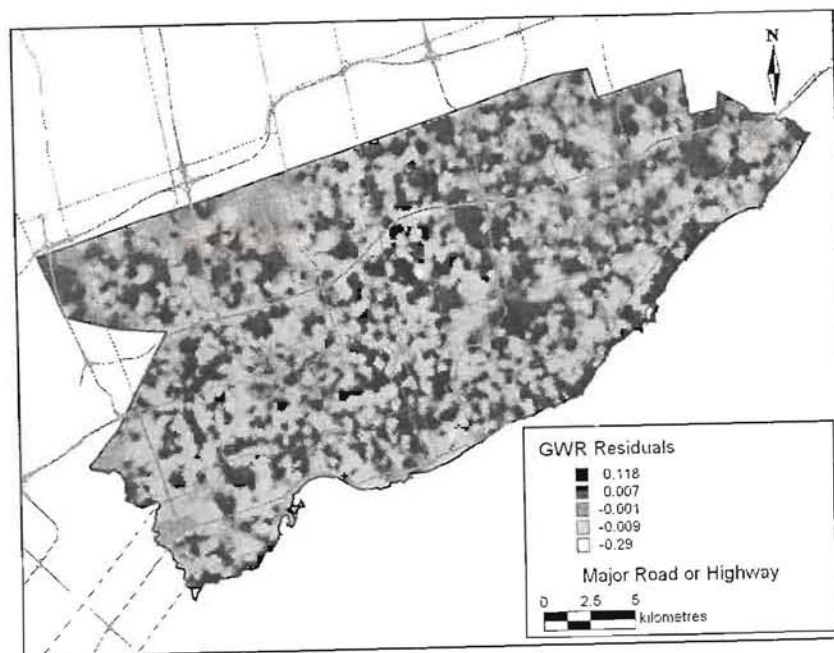
FIGURE 5 Pattern of Local R²S in the GWR Model

FIGURE 6 Surface of Residuals in the GWR Model

TABLE 3 Descriptive Statistics of GWR Coefficients Compared with 'Global' Model

	GOLS Coef.	25th Perc.	50th Perc.	75th Perc.
CONSTANT	1.43	2.10	2.80	3.47
LG_AREA	0.492	0.315	0.386	0.472
AGEPROP	-5.37E-04	-1.71E-03	-9.85E-04	-4.07E-04
SALEDAT1	1.36E-03	5.23E-04	1.30E-03	2.24E-03
LG_SIZE	0.196	0.158	0.200	0.250
QUALITY	0.049	0.013	0.027	0.042
DOWNDIS	-1.11E-05	-1.58E-05	-2.76E-06	8.84E-06
DISMAL	-7.31E-06	-1.94E-05	1.26E-06	2.05E-05
LG_HHINC	0.359	0.033	0.102	0.183
PC_FOR	-0.121	-0.103	-0.040	0.013

of-fit is spatially biased. To determine this, it is necessary to view the spatial pattern of residuals and measure the degree of spatial autocorrelation in their distribution (Figure 6). From simply viewing the map of residuals in Figure 6, clustering of positive or negative residuals is not evident -- the MC numeric quantity of spatial autocorrelation is not significantly different from zero. Thus by allowing the regression coefficients to vary over space, the GWR model has generated estimates with independent spatial error terms, the first of the models to do so.

In addition to the model's accuracy and the independence of the error terms, the GWR model can also be used to examine spatial heterogeneity of the regression coefficients. It would be useful to be able to determine whether or not the spatial pattern of the regression coefficients are significantly different from the 'global' coefficient, or if they are part of a random distribution. Unfortunately, there is no consensus in the literature on the most appropriate way of doing this (Fotheringham and Brunson 2004). So, a combination of maps (such as Figure 7), descriptive statistics and a measure of spatial autocorrelation has to be used to show that patterns appear to be non-random.

The table of GWR coefficients (Table 3) describes the numeric distribution of each variable's coefficient surface of which Figure 7 is an example. Each coefficient surface is significantly autocorrelated (by measure of Moran's Coefficient) and has discernable visual patterns (Wheeler and Tiefelsdorf 2005). Furthermore, the 'global' coefficients are not framed by the 25th and 75th percentiles in every case, indicating that in many locations GWR coefficients do not resemble their 'global' counterparts and may be considered not only nonstationary, but in some instances counterintuitive. For example, the coefficients for variables such as QUALITY and LG_HHINC should always be positive since better quality or more income would surely increase the price paid for a house in any neighbourhood, but they are not always so. On the other hand, it is intuitively acceptable that some of the regression coefficients for PC_FOR are negative and others positive, indicating that this variable may be associated with high property values in some neighbourhoods and low property values in others.

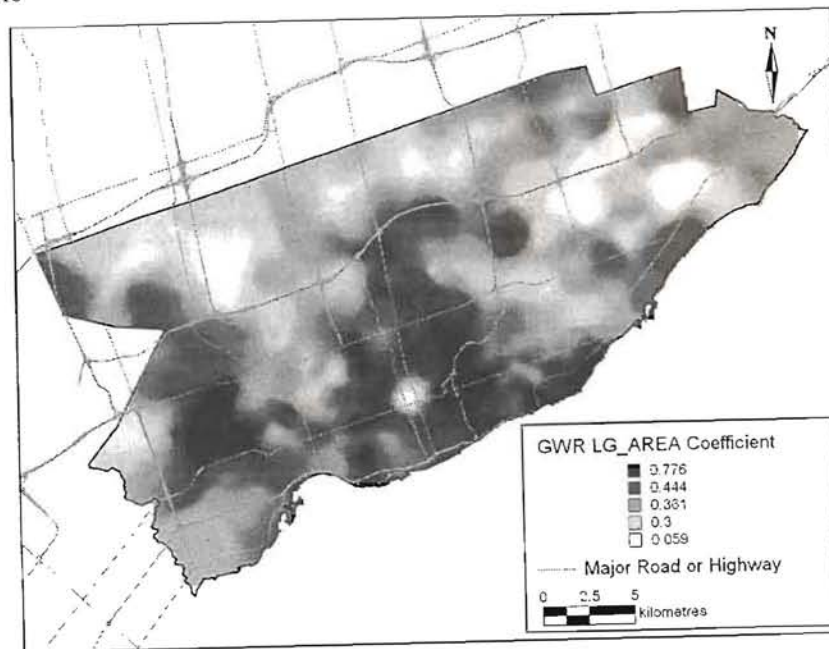


FIGURE 7 Surface of the GWR LG_AREA Coefficient

Moving Window Regression (MWR)

A MWR is essentially a special case of a GWR when W_i in Equation 5 is the $m \times m$ identity matrix, and m is the number of observations in the local regression. Thus (5) simplifies to the regular OLS regression equation:

$$B_i = (X_i^T X_i)^{-1} X_i^T P_i \quad (6)$$

where X_i [$m \times k$] and P_i [$m \times 1$] are subsets of the data pertaining to the local observations for regression i . For the purposes of comparison, m has been set at 274 for the GWR model – thus, each of the 19,007 local regressions consists of an OLS regression on observation i and its nearest 274 neighbours.

MWR analysis was performed using the same nine explanatory variables as before. Overall goodness-of-fit is high, with a pseudo- R^2 of 0.903, only 1.6 percentage points less than the GWR model, but 2.5 percentage points better than the SAR model. Also, the SSE of the model is quite low at 63.3 – not as low as the GWR residuals, but better than the SAR model. The MC for MWR residuals is 0.008 – reflecting in summary form an extremely low level of spatial clustering. Descriptive statistics for the coefficients estimated through MWR indicate that except for a few maximum and minimum values, the numeric distributions are quite similar to the GWR coefficients. The slight difference in results between the GWR and MWR models is evidently related to the weighting utilized in the former.

TABLE 4 Comparisons of R^2 's, the Sum-of-Squared Errors (SSE), and MCs

	R-Square	SSE	MC (raw)	MC Z-score
GOLS	0.831	109.9	0.24	294.3
SAR	0.878	79.2	0.092	113.9
GWR	0.919	52.7	-6.74E-06	0.056
MWR	0.903	63.3	0.008	10.2

Conclusion

This paper provides a comparison between four methods of 'global' and localized regression models for the purposes of residential property valuation in the City of Toronto. The models range from commonly applied methods for hedonic modelling, such as ordinary least squares regression, to more sophisticated mixed regressive-autoregressive and moving window regressions. The dataset is extremely large, containing 19,007 records of housing sales of various dwelling types taking place over a 12-month period from July 2000 to June 2001, and covering the entire metropolitan area. A parsimonious set of nine property attributes inclusive of measures of structure, neighbourhood, and accessibility to employment and service centres was used in the analysis. Significantly positive autocorrelation in the observed and predicted dependent variables resulting from traditional hedonic models provided the impetus to move towards spatially conscious, local models.

A comparison of goodness-of-fit and residual spatial autocorrelation is used to measure the relative effectiveness of the four models tested. The *goodness-of-fit* of a model is indicative of how well the estimated values correspond to those observed. Typically, the coefficient of determination (R^2) is used as a measure of overall goodness of fit. In the case of local regression, the R^2 must be substituted for a pseudo- R^2 . The GWR model with inverse distance weighting obtains the highest R^2 , 91.9%, while the simple regression on traditional structural variables scores quite low at 66.7% (Table 4).

The degree of *spatial autocorrelation* indicates how well the model addresses the spatial variation in the dependent variable. The MC has been calculated for each error vector (Table 4). It is apparent that spatial autocorrelation diminishes as the spatial complexity and accuracy of the model increases. The least spatially biased model is GWR (MC = 6.74E-06), while the GOLS model with no location based variables is extremely biased (MC = 0.24). Thus the GWR model may be regarded as the one which accounts best for the spatial variation in house prices.

Thus, the analysis suggests that as well as impacting a property's market value, the location of a property also determines the perceived values of other housing attributes. The local models clearly illustrate that regression coefficients show signs of non-stationarity, and that they vary in a haphazard manner not easily captured by higher-order functions such as trend surfaces (Yeates and Farber 2006). By allowing parameters to vary spatially, accuracy of the estimation of the dependent variable improves dramatically, and at the same time spatial biases diminish to nominal amounts.

One finding, however, is the presence of extreme coefficients, probably

resulting from local irregularities in variable distributions. In light of the unavailability of a robust statistical framework for GWR and MWR, irrational coefficients pose a major threat to the adoption of GWR by assessment authorities. Interestingly, locations with extreme coefficients are confined to specific neighbourhoods in the city, and estimates are as accurate there as they are for other parts of the city. It is the authors' contention that given more experimentation with different calibrations of the number of near neighbours used in each local regression, the issue of extreme coefficients could be mitigated with very little impact on overall model accuracy.

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