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THE MULTICRITERIA CHOICE PROBLEM AND PLAN EVALUATION: A TEST OF A METHOD[1]

Bryan H. Massam
York University

Robert A. Wolfe
University of Toronto

Introduction

Considerable interest is shown by academics, planners, politicians and the public in the topic of the generation and the evaluation of alternate plans, and the selection of a best alternative. Interest ranges from the theoretical treatment of abstract optimization problems to practical resolution of conflict situations. All recognize that the problem of defining and searching for the best alternative is complex, because multiple criteria and goals must be considered. Not infrequently, many individuals and groups will be involved in the determination of the criteria and goals; the search process which involves the collection of information may be lengthy and costly; opinions and preferences may shift during the study, and conflicts can arise. The scale of the study and the types of plans considered as well as the time horizon over which the search is made, the implementation of the plan undertaken, and the impacts evaluated all serve to complicate the problem further.

We might argue that, given such a complex decision-making environment, it is most unlikely that any formal method could be developed to identify a best alternative. Nevertheless, attempts are made to structure the decision-making procedure and to incorporate measurements of impacts into the decision process. A variety of procedures have been used by geographers, planners and applied mathematicians, among others, to help in the evaluation of alternatives and the selection of one from a feasible set.

The classification of procedures shown in Table 1 is an attempt to summarize the range of formal methods currently available. The overlay and cost surface methods have been developed specifically to solve location choice problems, whereas the other methods have more general application to multicriteria choice problems.

In certain cases the procedure can be applied to a set of data for alternate plans and a "best" plan determined, for example, by using the overlay or the cost surface methods or mathematical programming. It should be noted that under certain conditions a unique solution may not exist. Other procedures (for example, the matrix methods) typically offer a summary of the information without an explicit suggestion as to which of the alternatives is the best. It is our contention that a useful contribution to plan evaluation can be made if a formal method is incorporated into the social-political milieu within which choice is undertaken, and the formal method should not only summarize data on the variety of criteria and impacts believed to be important but present the information in such a way that the relative merits of the contending plans are shown. Thus, rather than present the decision-maker with a best solution, it is our aim to present a map of the alternatives. The dimensions of the space within which the map is produced will preserve

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Table 1

A CLASSIFICATION OF PROCEDURES FOR SOLVING MULTICRITERIA CHOICE PROBLEMS

A Overlay	B Cost surfaces over a study area	C Matrix methods	D Mathematical Programming/Analysis
1. Manual	Turner & Hausmanis [24]	1. Check list	Thiriez & Zionts [21]
McHarg [11]	Goodchild [2]	Leopold *et al*. [8]	Zeleny [26]
2. Computer Troughton & Newkirk [23] Potts [16] Owens [15]	Nemett [12]	2. Lexicographic ordering Holmes [4;5;6] Kettle & Whitbread [7] Nowlan [14]	
		3. Powering Ross [18]	
		4. Sequential Sorenson [20] Fischer & Davies [1]	
		5. Goals achievement Hill [3]	
		6. Planning budget sheet Lichfield *et al*. [9]	

the variety of information pertaining to the impacts of each plan on each criterion.

A New Procedure

The procedure offered here is an extension of the one developed by Roy [19] for selecting an alternative from a feasible set, given information in the form of an impact matrix, which displays the impact of each alternative on a set of criteria. This matrix is sometimes referred to as a project effect matrix or an evaluation table.

Given a set of plans ($p_1, \ldots, P_m$) and a set of criteria ($Q_1, \ldots, Q_n$) on which each plan receives a rating R_{ij} for plan i on criterion j, and a set of criteria weights $W_1, \ldots, W_n$, three types of sets can be defined:

a concordance set C (i,i´)

a discordance set D (i,i´)

a tie set T (i,i´)

C (i,i´) = {j such that, w.r.t. criterion j i > i´}

D (i,i´) = {j such that, w.r.t. criterion j i < i´}

T (i,i´) = {j such that, w.r.t. criterion j i = i´}

From the concordance and tie sets the following concordance index can be calculated for a pair of plans i and i´,

$$c_{ii'} = \frac{\sum_{j\varepsilon C(i,i')} W_j + \frac{1}{2}\sum_{j\varepsilon T(i,i')} W_j}{\sum_{j=1}^{m} W_j}$$

The concordance index measures the degree of unanimity over the criteria of a preference for plan i over plan i´.

Values for $c_{ii'}$ range from 1 to zero; when plan i dominates i´ for all criteria, $c_{ii'}$ = 1.0. A concordance matrix summarized the values for all pairs of plans. A sample matrix is shown in Figure 1. Because we have incorporated ties into the calculation of the index, values in complementary cells sum to unity. In Roy's method this would not occur, as a different formula is used to calculate cii´.

The style of this matrix is similar to that used in preference analysis and it can easily be converted to a symmetric dissimilarity matrix using the following transformation,

$$V_{ii'} = \left| 0.5 - c_{ii'} \right|$$

where $V_{ii'}$ is a measure of dissimilarity between two plans i and i´. This is shown by Figure 2 for the example in Figure 1. A value of .5 suggests the two alternatives are dissimilar and one clearly dominates the other; as the value approaches zero so the dominance of one by another diminishes.

Using multi-dimensional scaling, it is possible to examine such a matrix and produce a map showing the relative positions of the alternatives. To measure the agreement between the map and the values in the matrix Kruskal's stress coefficient can be used. If this is zero

Figure 1

SAMPLE CONCORDANCE MATRIX

PLANS	1	2	3	4
1	0	.4	.5	.1
2	.6	0	0	.7
3	.5	1.0	0	.2
4	.9	.3	.8	0

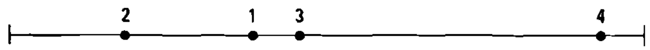
Figure 2

DISSIMILARITY MATRIX

PLANS	1	2	3	4
1	0	.1	0	.4
2	.1	0	.5	.2
3	0	.5	0	.3
4	.4	.2	.3	0

Figure 3

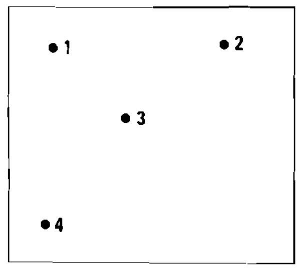
HYPOTHETICAL ONE - DIMENSION MAP



(PLANS 1,2,3,4)

Figure 4

HYPOTHETICAL TWO - DIMENSION MAP



(PLANS 1,2,3,4)

Table 2

ORIGINAL DATA MATRIX

	1	2	3	4	5	6	7
1	3.000	4.000	8.000	7.000	3.000	5.000	1.000
2	0.000	20.000	300.000	270.000	0.000	70.000	0.000
3	6.000	6.000	1.000	2.000	5.000	4.000	7.000
4	4.000	4.000	1.000	1.000	6.000	4.000	7.000
5	4.000	3.000	2.000	2.000	4.000	6.000	9.000
6	4.000	6.000	32.000	30.000	5.000	20.000	20.000
7	450.000	550.000	780.000	195.000	395.000	650.000	365.000
8	8.000	7.000	5.000	5.000	8.000	7.000	4.000

then the map is a perfect representation of the values in the matrix. A one-dimension map of the style shown in Figure 3 will have a higher stress coefficient than a two-dimension map of the style shown in Figure 4.

Such maps can be used to identify similar plans, or for ordering the plans. Also, it might be possible to correlate the distribution of the alternatives with attributes of each plan. Further, by altering the values in the initial matrix we can examine the stability of the maps. It is sometimes the case that when the concordance matrix is formed, some values of the project effect matrix overrate others by only a small margin. Possibly these cases should be treated as if the plans were ties on a particular criterion.

The computer routine [10] used for the analysis includes an option referred to as the just noticeable difference (JND), which allows the user to define differences between scores of alternate plans on a criterion that is to be treated as insignificant. The following index is calculated: [(s/l)100], where s is the smaller score and l the larger score. This index is compared to a critical value provided by the user. If the index is greater than the critical value the two plans are considered to have equivalent impacts on the particular criterion. For example, if the critical value is 100, then all differences between scores will be used in the calculation of the concordance index; as the critical value approaches zero, larger differences between the scores will be treated as insignificant.

A Practical Application

To illustrate the procedure outlined above we will examine a plan evaluation problem presented by Nijkamp [13], which he analyzed using Roy's method and data for a reclamation scheme in the IJsselmeer. Seven alternate plans were defined and each was examined using eight criteria. For more than thirty years, reclamation projects have been in operation in the Zuider Zee to protect inland areas, to offer a secure water supply and to make available space for urban and agricultural expansion. The data used by Nijkamp are for one of the final phases in the Markerwaard area. The impact matrix is shown in Table 2. Nijkamp uses two weighting schemes; the first shows a strong preference for environmental preservation and regional well-being, while the second emphasizes production potential and employment. In this study a third scheme, of equal importance for each criterion, will also be used. Table 3 summarizes the weights. The negative signs indicate cases where an increase in the value decreases the impact.

Table 3

WEIGHTING SCHEMES

Criteria*	Environment	Production	Equality
1	-.234	-.087	-.125
2	.118	.130	.125
3	-.118	-.174	-.125
4	-.118	-.044	-.125
5	-.059	-.044	-.125
6	.059	.217	.125
7	-.059	-.217	-.125
8	-.234	-.087	-.125

*Full definitions of the criteria are given in Nijkamp [13].

Using three critical values (100, 80 and 50) for the just noticeable difference together with the three weighting schemes, a set of nine subproblems was produced. This is shown in Table 4.

Table 4

TYPOLOGY OF NINE SUBPROBLEMS

Just noticeable difference	Weights		
	Environment	Production	Equality
100	9	6	3
80	8	5	2
50	7	4	1

For each of the nine cases a concordance and a dissimilarity matrix was produced. The latter was analyzed using the multidimensional scaling routine MINISSA [17], and maps in one, two and three dimensions were produced. For each, Kruskal's stress coefficient was calculated. The formula for calculating the values in the concordance matrix is the one given earlier and takes into account tied values.

Roy's original method for determining a best alternative uses a concordance matrix and a discordance matrix. Values for the latter are calculated from information in the discordance set $D_{ii'}$, using the formula given in Nijkamp [13, p.11].

For any pair of plans i and i′ two indices are now available which can be summarized on two matrices, a concordance matrix and a discordance matrix. Values for each index range from 1.0 to zero; $c_{ii'} = 1.0$ and $d_{ii'} = 0$ when plan i dominates i′ for all the criteria. As it is unlikely that one alternative dominates all others for all the criteria, Roy suggests that in order to identify a dominant plan we have to accept concordance values which are less than 1.0 and discordance values which are greater than zero. Roy uses graph theory and different threshold limits for $c_{ii'}$ and $d_{ii'}$ to identify the dominant alternative. While his method allows one of the alternatives to be selected as the best, the method proposed in this paper allows us to classify all of the alternatives; the relative position of each of the alternatives can be seen. A summary of the multidimensional scaling analysis is given in Table 5.

With the exceptions of cases 6, 3, 4 and 8, the one-dimension maps give low stress coefficients which suggests that the map is a close representation of the values in the original dissimilarity matrix. Cases 2, 5 and 7 are probably degenerative, as the alternatives are divided into two groups which cluster at the ends of the scale.

Maps in three dimensions have almost insignificant stress coefficients but, unfortunately, are not particularly easy to visualize. A working compromise is offered by the two dimension maps which generally have very low stress coefficients and can be easily drawn. A comparison of the one-dimension scales is given in Figure 5 and Table 6.

While we might expect the two weighting schemes to give different maps, there is in fact a statistically significant correlation at the .05 level between cases 9 and 6; Spearman's correlation coefficient is .68. Neither case is correlated with the equality weight scheme in which all criteria are treated equally. For experimental purposes a low just noticeable difference of 50 was included to test the notion that this may cause maps produced under different weighting schemes to converge to a common pattern. Correlation coefficients among cases 7, 4 and 1 were not statistically significant.

Table 5

RESULTS OF MULTIDIMENSIONAL SCALING ANALYSIS:
KRUSKAL'S STRESS COEFFICIENT

Case	Dimensions 1	2	3
1	.001	.000	.000
2	.000*	.000	.000
3	.261	.001	.001
4	.254	.063	.002
5	.005*	.007	.001
6	.336	.143	.075
7	.001*	.001	.001
8	.237	.125	.086
9	.162	.001	.001

*Degenerate solutions

Figure 5

A COMPARISON OF ONE-DIMENSION SCALES

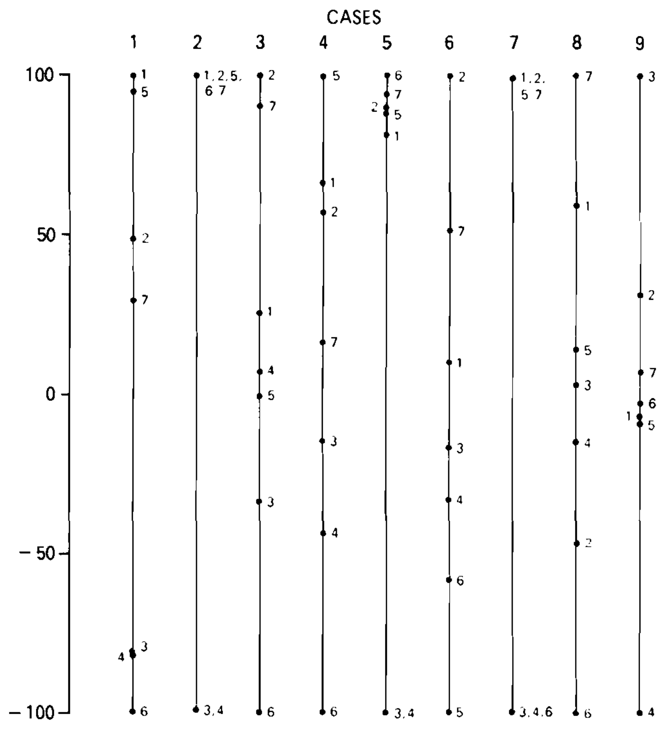


Table 6

ONE-DIMENSION SCALES

Cases	Alternatives 1	2	3	4	5	6	7
1	100.000	48.324	-80.924	-81.592	94.914	-100.000	29.581
2	99.999	99.998	-100.000	-99.999	99.999	100.000	99.999
3	25.111	100.000	-34.159	7.282	-0.279	-100.000	90.276
4	66.263	57.070	-15.331	-44.209	100.000	-100.000	16.361
5	81.894	90.071	-100.000	-99.999	88.456	100.000	94.097
6	10.094	100.000	-17.077	-33.777	-100.000	-58.698	51.889
7	99.749	99.374	-99.840	-100.000	100.000	-99.379	99.133
8	59.190	-47.029	3.691	-15.337	14.351	-100.000	100.000
9	-7.048	31.757	100.000	-100.000	-9.662	-3.509	7.412

The next task is to attempt to assign labels to the ends of the one-dimension maps. Nijkamp [13, p. 23] suggests that when the concordance value is relaxed from 1.0 to .700, alternatives 7, 3 and 2 warrant closer inspection. By counting the number of times the discordance index falls below a threshold of .150 we find that alternative 7 could be eliminated and alternative 3 scores slightly better than 2. This refers to the environmental weighting scheme. For the production weighting and using threshold values of .600 and .200, alternatives 3, 5 and 7 emerge as deserving closer study. Using a count of the number of times a plan satisfies the threshold value, Nijkamp claims that alternative 3 is the most preferred. In both cases, then, this appears to be the most appropriate choice. In Figure 5 it would therefore seem appropriate to label the high scores for cases 9, 6 and 4 as most preferred.

For the three degenerate solutions (2, 5 and 7), alternative 3 is in the cluster at the low end of the scale, and for cases 1 and 3 it tends towards the lower end, while the case 8 it occupies a central position. The task of assigning labels to the ends of the scale is not based upon any formal procedure; hence it tends to be subject to the whims of the analyst's vision and vacabulary. Probably a more satisfactory procedure is to present the maps for discussion and debate without labelling the axes or including numerical scores, remembering that the purpose of this work is to present a map of the alternative plans in a space which is as consistent as possible with the data originally given in the impact matrix. With this in mind, in this particular problem we should use the two-dimension maps, as they exhibit low stress coefficients. Figure 6 is a comparison of cases 6 and 9 in a two-dimension space; it is included to illustrate the differences between the two weighting schemes suggested by Nijkamp. For comparitive purposes a plot of case 3 is included on the same map. The latter distribution shows three clusters, whereas cases 6 and 9 show a dispersed distribution. Perhaps we could argue that the most preferred alternative according to Nijkamp - number 3 - occupies a similar location in both cases 6 and 9. This is in sharp contrast to its location in a cluster with alternative 6. A measure of the similarity between cases 6 and 9 was undertaken using Tobler's [22] method and a recent method of Wolfe [25]. The results are given in Table 7.

Table 7

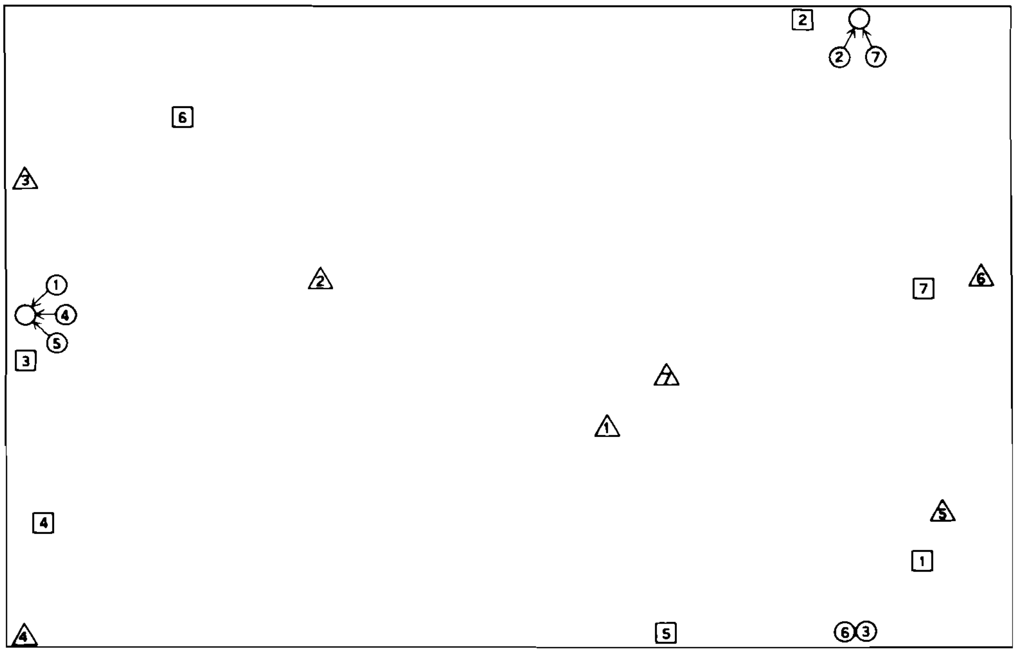
A COMPARISON OF CASES 6 AND 9:
TWO-DIMENSION MAPS

		Tobler	Wolfe
Spatial correlation coefficient		.94	---
Scaling of case 9 to fit 6		.828	1.24
Rotation: degrees		4.00	5.0
Translation	x	---	17.21
	y	---	12.61

The maps appear to be very similar. If further work on the stability of the maps to a variety of changes to the weighting of the criteria is to be undertaken then it will be necessary to develop statistical measures for comparing maps. Research is needed in this field.

Figure 6

A COMPARISON OF CASES 3, 6 AND 9. TWO - DIMENSION MAP



① Case 3
[1] Case 6
△1 Case 9

Conclusion

The final section of this paper will offer a comparison of the results produced by Nijkamp and those generated by the new procedure. Also, we will offer comments about the utility of the procedure for tackling plan evaluation problems which involve multiple criteria.

A comparison of our results with those of Nijkamp is given in Table 8. Nijkamp is only able to identify the order of the two alternatives which rank highly. For both weighting schemes, environmental and production, the best alternative is plan 3, and the second best is plan 2. The relative positions of the other five plans are not identified. The results produced by the new method not only give an order for all seven plans but locate the plans on an interval scale (see Figure 5). The comparisons on Table 8 shows that plan 2 appears at the end of the scale under the equal and production weighting schemes, and in second position for the environmental weighting scheme. Only in one case - environmental weighting - is plan 3 at the end and in accord with Nijkamp's results. The scaling results shown are derived using a JND of 100. Our results suggest that plans 2 and 7 deserve attention as being better than most of the others. The recommendation we would make would be to reject plans 4, 5 and 6, as they are consistently at the lower end of the scale, and to focus attention on plans 2, 7, 1 and 3, as they appear to be the better ones. The next step could be to analyze a matrix with impacts for only these plans, perhaps to include a more detailed list of criteria.

Table 8

A COMPARISON OF RESULTS

Weighting order	Nijkamp		New method*		
	A	B	A	B	C
1	3	3	3	2	2
2	2	2	2	7	7
3			7	1	1
4			6	3	4
5			1	4	5
6			5	6	3
7			4	5	6

* Interval values are shown on Figure 5.
A Environmental criteria; B Production criteria; C All criteria equal weight.

The apparent ease with which Roy's method can be used to identify a best alternative should not ignore the fact that in order to do so it is necessary to relax the concordance and discordance indices from 1.0 and zero. This inevitably introduces a problem into both the interpretation of the significance of the final results and the selection of a single alternative. It is our contention that the new method, because it allows us to identify the relative positions of all the alternatives on both an ordinal and an interval scale, serves to provide guidance to those concerned with rejecting alternatives and identifying a best one. Further, the new method is sufficiently flexible to allow changes to the JND to be made and different weights can be assigned to each criterion. The effects of these changes can be shown by examining the new ordinal and interval scales. Perhaps these scales

are stable under a variety of different weighting schemes, or perhaps a slight change to a weighting scheme or the value for the JND radically alters the position of alternatives on the scales. This information is surely of interest in the evaluation of the alternatives.

At this time the method is being applied to hypothetical plan evaluation exercises. It would appear to merit close attention by those involved in the analysis of impact matrices as a tool for organizing data in order to compare plans.

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