

# **A Bi-Regional Economic Impact Model for the Province of British Columbia: A Social Accounting Matrix Approach \***

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Local (sub-state or sub-province) economic impact studies have relied heavily on the simple economic base method because it is less demanding in terms of data requirements than more sophisticated general equilibrium methods, specifically macroeconomic models estimated from time-series data using econometric methods, input-output (IO) models, social accounting matrices (SAMs) and computable general equilibrium (CGE) models.

This paper outlines a prototype SAM model for British Columbia and its uses for regional and provincial economic impact analysis. The SAMs reported in the paper are also the preliminary results in a bi-regional CGE-building project.

The main contributions of the study are as follows. First, it shows the feasibility of developing a database at the regional level within the province using locally specific survey and secondary transactions data in combination with provincial and national data (the so-called 'hybrid' approach to model building) as an advance on the exclusive use at the regional level of transactions data from other jurisdictions (the 'synthetic' approach). Not excessively demanding in terms of data require-

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ments, the approach could be implemented for regional configurations in BC other than the one adopted in this study, or for regions of other provinces. Second, the model is constructed for two regions of the province as well as the province as a whole, an advance on single-region models that cannot allow for inter-regional feedback effects in the spending stream and do not permit simulations of simultaneous impact in different regions and the province overall. To date, as indicated in detail below, regional macroeconomic, IO, SAM and CGE modeling in Canada has been confined to (i) provincial/territorial-level models and (ii) single-region models at the sub-provincial or sub-territorial level, in some cases employing merely the synthetic approach to model building.

The more sophisticated approach to general equilibrium modeling has been extended from the national to the provincial and territorial levels with development by governments of macroeconomic, IO and CGE models at those levels (for a Quebec example, see Decaluwé et al 2003, 2004), by Statistics Canada of its inter-provincial IO model and by researchers who have constructed province-wide CGE models to investigate particular issues (e.g. Constantino and Percy 1988; Percy and Constantino 1989; Binkley et al 1994; Alavalapati et al 1996; Sandhu and Percy 1997; Sandhu 2000). At the sub-provincial or sub-territorial level, while there are examples of off-the-shelf impact models that are used in selected sectors (e.g. TEAM for the tourism sector (Conference Board of Canada 2005)), there is no ready-made application system such as IMPLAN in the US (Minnesota IMPLAN Group Inc 1999) or YourPlace-IO in Australia (NIEIR, undated) that can be used for general-purpose impact evaluation at that level. All such canned sub-provincial or sub-territorial models, however, are synthetic and rely on technical coefficients from higher jurisdictions. Sub-provincial/territorial general equilibrium modeling based on at least some locally specific transactions data has been limited in Canada to a few pioneering studies (e.g. Davis 1976; Duhaime 1991; Frechette et al 1992; Lemelin 1993, 1994, 1998; Siemens and Kulshreshtha 1995; Patriquin et al 2002, 2003), and each is for a single region alone.

In BC, province-wide impacts of policy initiatives or changes in trading conditions can be gauged through the provincial econometric and IO models maintained by the Ministry of Finance and Corporate Relations (Government of BC 1999, undated) and through selected purpose-built CGE models as cited above. Estimates of impact at the community level can be approximated using available economic base employment multipliers for 63 local communities (Horne 2004), although these are based on province-level technical coefficients from the BC IO model (BCIOM). At the regional level between province and community, there are dated estimates of expanded Keynesian income multipliers that incorporate transactions information from an early IO model of the Vancouver economy (Davis 1986), and extensions of Horne's work to selected Regional Districts (Synergy Management Group 2004). However, there is no model for evaluating impacts on broad regions within the province that is not based exclusively on use of the technical coefficients of another jurisdiction. Moreover, apart from the Vancouver model that contains data of the early 1970s, there been no regional macroeconomic, IO, SAM or CGE model for even a single region of BC, let alone both a study region and the rest of the Province.

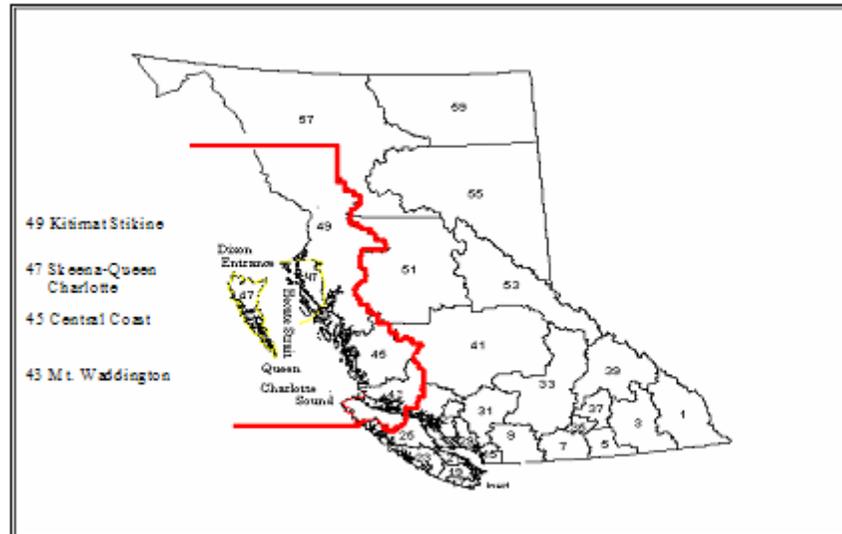


FIGURE 1 Queen Charlotte Basin Region of BC

In what follows we outline the key issues involved in the construction of a two-region SAM using available local transactions data for (i) the coastal region of BC surrounding the Queen Charlotte Basin (QCB) that covers Hecate Strait and Queen Charlotte Sound (Figure 1) and (ii) the rest of the Province. We also show selected direct and indirect output, GDP, household income, employment and government revenue impact ratios by sector at the regional and provincial levels as generated from the model. These are the ratios that can be used to gauge the impact on total output, GDP, household income, employment and government revenue of an exogenous change of one unit of output in a given sector. The QCB region (Figure 1) comprises the four regional districts (RDs) of Mount Waddington (RD 43), Central Coast (RD 45), Skeena-Queen Charlotte (RD 47) and Kitimat-Stikine (RD 49), an area that includes many of the communities that have felt the stress of recent difficulties in the resource sectors of BC.

The next section of the paper discusses construction of the model. The third section summarizes selected sectoral impact ratios generated by the model, and conclusions are drawn in the final section. The Appendices contain the estimated SAMs for BC, the QCB region and the rest of the province as well as an algebraic description of the bi-regional model.

## SAM Construction

A SAM depicts the circular flows of payments and income in matrix form with expenditures in columns and receipts in rows (a macro SAM that abstracts for simplicity from disaggregation of main accounts is shown in Table 1). The flow of intermediate commodities between producing sectors of the economy as cap-

**TABLE 1 Macro Social Accounting Matrix**

	Factors				Institutions			Rest of World	Total
	Industries	Commod.	Labour	Capital	Enterp.	Households	Gov't		
Industries		Domestic supply (make matrix)							Total sales
Commod.	Interm. use (use matrix)				Household consump.	Gov't consump.	Public & private invest.	Exports	Total demand
Labour									Labour income
Capital	GDP at factor cost								Capital income
Enterprises			Gross profits		Personal transfers to corp.	Transfers to corp.			Enterprise income
Households			Gross labour income	Distributed profits		Transfers to households		Remit. from ROW	Househ'd income
Gov't	Indirect taxes on industries	Indirect taxes on final demand	Social insur'ce contribs	Direct corp. tax	Direct pers. tax			Direct tax from ROW	Gov't income
Capital Account				Retained earnings	Personal savings	Gov't savings		Capital transfers from ROW	Total savings
Rest of World		Imports			Transfers to non-residents	Transfers to non-residents	Capital transfers to ROW		ROW exchange payments
Total	Total costs	Total supply	Labour comp'n income	Capital income	Enterprise expend.	Household expend.	Gov't expend.	Total invest.	ROW exchange earnings

Note: Not all cells are required for the current analysis. For details, see Appendix A, Table A1, note 2.

tured in input-output tables is shown in the use and make (or production transactions) matrices of the SAM. Following the design of the Canadian IO table, the use and make matrices in this study are rectangular (comprising 21 commodities and 18 sectors) in contrast to the square matrices in, for example, US tables. Other accounts in the SAM are grouped as they relate to factors of production (labour and capital), current expenditures and receipts of institutions (enterprises, households, governments), financial capital flows (investment spending and saving) and the rest of the world. As the sum of all expenditures in a given account must equal the sum of all receipts or income in that account, row and column sums of corresponding accounts must be equal.

While SAMs remain fixed-price general equilibrium models like IO models (as opposed to CGE models that allow for market clearing through flexible prices), they can be used to estimate not only aggregate and sectoral impacts of exogenous changes on the economy as with IO models, but also the distribution of impacts within institutional categories. In the current model the focus is on differential revenue impacts on federal and provincial levels of government.

SAMs were estimated in the study for each of the years 1997-99, although results are shown for 1999 only. The hybrid approach described earlier in the paper involves use of local primary and secondary data where available for estimation of input-output and cross-boundary trading relations in key sectors of the economy. The core elements of the model are separate SAMs for the province as a whole and the QCB region, the rest of the province being modeled as the residual. The SAMs for the Province of BC, the QCB region and the rest of BC for 1999 are shown in summary form in the Appendix with several commodities and sectors compressed to avoid excessive data proliferation. As displayed, there are 6 commodities and 6 sectors, including 5 key resource sectors of the QCB region.

### **Provincial SAM**

Data for the provincial SAM are taken directly from the BCIOM (Statistics Canada 2003a) and the provincial economic accounts (Statistics Canada 2002) with separate estimation procedures for disaggregating the manufacturing sector in order to highlight the pattern of specialization in the QCB region. Thus, sectoral aggregation in the provincial SAM is based on the aggregation level (small) of the BCIOM, except that the manufacturing sector is disaggregated to highlight seafood processing, wood products, and paper manufacture separately from the rest of manufacturing. Instead of relying solely on the technical coefficients that are available at the national level for disaggregating the manufacturing sector, the approach builds data from the bottom up using provincial information as available. This approach is designed to reflect characteristics of the different manufacturing industries that are unique to British Columbia.

#### *Use and make matrices*

Disaggregation of the manufacturing sector in the use matrix involves estimation of (a) provincial output in each sub-sector by allocating total manufacturing output proportionally to provincial manufacturing shipments (Statistics Canada 1999) and (b) total intermediate commodity use in each sub-sector calculated as the value of total output (= the value of total input) less GDP at factor cost and net indirect taxes (for estimation of GDP at factor cost and indirect taxes, see below).<sup>1</sup> Intermediate use is divided into materials and energy use by sub-sector from principal statistics of manufacturing industries in BC (Statistics Canada 1999) and commodities are allocated to materials and energy use according to sub-sector use in the large aggregation of industries in the Canadian IO table (Statistics Canada 2003a). In the make matrix, allocation of commodities to the manufacturing sub-sectors not shown in the BCIOM treats the output of these sub-sectors as single commodities.

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1. Shipments rather than experienced labour force are chosen as the proxy for output in manufacturing sub-sectors. This is because shipments data are available for each year whereas the labour force data are available for census years only. Where shipments data are not available at other points in the analysis, experienced labour force is used as the output proxy.

*Factor accounts*

GDP at factor cost for sub-sectors of manufacturing is estimated as GDP at basic price (Statistics Canada 2003b) adjusted for net indirect taxes on factors of production (e.g. property taxes, payroll taxes, training subsidies) using the adjustment ratio by sub-sector in the large aggregation Canadian IO table. GDP at factor cost in manufacturing sub-sectors is then allocated to labour and capital accounts using labour income for each sub-sector (Statistics Canada 2003a) and allocating the balance of GDP at factor cost to the capital account.

*Government revenues*

The allocation of indirect taxes and subsidies to each sub-sector of manufacturing is based on the proportion in the provincial manufacturing sector (BCIOM). Indirect taxes are separated into taxes on factors of production and taxes on products using ratios in the provincial economic accounts. Indirect taxes on factors and products are then allocated to federal, provincial and local government accounts on the basis of the respective government shares in the provincial economic accounts. Direct taxes on corporations are allocated across industries on the basis of the provincial effective tax rate. Direct personal taxes on households are allocated across industries on the basis of estimated effective rates of taxation and mean income by industry. Direct taxes are distributed across levels of government in accordance with data in the provincial economic accounts.

**Regional SAM (QCB Region)**

The hybrid approach to calculation of the QCB SAM involves assembling as much region-specific data as possible and using provincial coefficients otherwise. Below we describe in turn procedures used for estimating use and factor matrices, the make matrix, final demand matrices, and government revenue accounts.

*Use matrix and factor accounts*

Estimation of the accounts in the intermediate use and factor matrices proceeds in three steps involving calculation of total use/output, intermediate commodity use and factor use (GDP at factor cost). Specific procedures vary by industry according to local data availability.

In primary sectors, output in the forestry and logging industry is based on production data for both the province and QCB region provided by the BC Ministry of Forests. Intermediate commodity use is based on provincial use (BCIOM). Labour income is calculated from GDP at factor cost (BCIOM) and data on labour income per thousand cubic meters of logs harvested in the BC coastal region (Price Waterhouse 2000). Remaining factor income in forestry and logging (GDP at

factor cost less labour income) is allocated to the capital account. Output of the fishing, hunting and trapping industry is based on the ratio of the regional to the provincial experienced labour force in that industry (BC Stats undated). Intermediate and factor uses are then allocated according to the proportions for fishing, hunting and trapping in the BCIOM. The same procedure is used for other primary sectors.

In the manufacturing sector, the total value of seafood output in the region is calculated on the basis of the proportion of the provincial wholesale value of seafood products produced in the region using data supplied by the BC Ministry of Agriculture, Food and Fisheries. Along with use of published sub-provincial principal statistics for manufacturing (BC Stats 2001), a local survey of seafood processors was conducted to generate estimates of the proportionate use of material, energy and labour inputs. Material and energy inputs are then spread across commodities in accordance with proportions in the seafood processing industry in the provincial SAM. Remaining factor incomes after adjustments for indirect taxes are allocated to the capital account.

In wood products and paper manufacturing, regional output is calculated using data on regional and provincial production supplied by the Ministry of Forests. Intermediate commodity use is based on ratios in the provincial SAM with adjustments for log inputs as supplied by the BC Ministry of Forests. These adjustments are important to reflect differences in production functions between coastal and interior wood and paper manufacturing industries. GDP at factor cost for each sub-sector is based on provincial ratios, with labour income separated from capital income on the basis of labour cost as a proportion of the value of output in the respective coastal industries (Price Waterhouse 2000). The rest of the manufacturing sector is estimated as the residual of total manufacturing minus seafood products, wood products and paper manufacturing calculated as above.<sup>2</sup>

For other sectors not shown separately in this paper (utilities, construction and services), regional output is estimated in terms of Census 2001 sectoral shares of the experienced labour force at the provincial level (BC Stats undated), with coefficients from the BCIOM used to estimate intermediate and factor use.<sup>3</sup>

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2. Total use in the manufacturing sector in the QCB region is estimated from the proportion of the value of total manufacturing shipments in the province originating in the QCB region (BC Stats 2001). Inputs used in the manufacturing sector as a whole are based on their respective coefficients in the BCIOM.

3. While experienced labour force data are based on place of residence rather than place of work, this is unlikely to be problematic because there is no significant commuting between the QCB region and the rest of the province. Use of Census 2001 labour force data (2000 operational data) as the proxy for 1999 output involves a minimal temporal discrepancy and therefore minimizes any difficulty in proxy use arising from volatility in sectoral activity over time.

*Make matrix*

The make matrix for the region is constructed by assuming that each industry in the region produces the same proportionate mix of output as at the provincial level.

*Final demand matrices*

Household consumption in the QCB region is calculated for each commodity from per capita expenditure on each commodity at the provincial level. Government current expenditures and public and private capital expenditures are estimated on the basis of their respective proportions of provincial domestic output. Estimation of trade flows across regional boundaries, always a significant issue in regional modeling, is addressed by calculating total regional net trade by commodity as the difference between total regional demand and total regional output. Regional trade with the rest of Canada (ROC) and the rest of the world (ROW) is based on the provincial ratio of net trade with ROC and ROW to domestic output. Regional trade with ROC and ROW is then allocated to exports and imports on the basis of provincial proportions of trade with ROC and ROW. The difference between total regional net trade and regional trade with ROC and ROW is allocated as net regional trade with the rest of BC, except that in the seafood processing and forestry and logging sectors, use was made of superior regional export data from, respectively, a survey of seafood processors in the region and information from the BC Ministry of Forests on log production and exports to ROC and ROW.

*Government revenues*

Indirect taxes for each sector are based on the ratio of indirect taxes to total output of the respective sectors in the provincial SAM. Using provincial ratios, indirect taxes are then separated into taxes on factors of production and taxes on products and spread over the different level of government. Direct corporation and personal income tax receipts are based on effective tax rates and, for personal taxes, mean income by industry, assuming that productivity is the same at regional and provincial levels.

## **Sectoral Impact Ratios**

Table 2 displays the impact ratios derived from the SAMs for key sectors of the QCB economy.

Monetary impacts are measured in millions of dollars, except for taxes that are measured in thousands of dollars. The ratios show the changes in total output and the changes in direct and total GDP, household income, employment (measured in person years) and government revenue in the QCB region, the rest of the province and the province as a whole associated with an initial unit change in output

**TABLE 2 Selected Industry Impact Ratios**

	Exogenous \$1m output change in QCB region					Exogenous \$1m output change in RoBC region				
	Forestry and logging	Fishing, hunting and trapping	Seafood products	Wood products	Paper Products	Forestry and logging	Fishing, hunting and trapping	Seafood products	Wood products	Paper products
<b>Output</b>										
QCB Total	1.611	1.459	1.684	1.716	1.675	0.019	0.002	0.052	0.062	0.027
RoBC Total	0.162	0.140	0.114	0.104	0.125	1.683	1.493	1.690	1.796	1.753
BC Total	1.774	1.599	1.799	1.820	1.800	1.702	1.495	1.741	1.858	1.781
<b>GDP (factor cost)</b>										
Direct	0.448	0.406	0.297	0.425	0.289	0.451	0.406	0.297	0.425	0.289
QCB Total	0.661	0.569	0.567	0.715	0.568	0.008	0.001	0.020	0.026	0.011
RoBC Total	0.076	0.069	0.057	0.050	0.062	0.743	0.630	0.602	0.778	0.635
BC Total	0.736	0.638	0.624	0.765	0.630	0.751	0.631	0.623	0.803	0.647
<b>Household Income</b>										
Direct	0.339	0.294	0.172	0.220	0.229	0.235	0.294	0.172	0.212	0.199
QCB Total	0.492	0.398	0.358	0.421	0.402	0.006	0.000	0.014	0.019	0.008
RoBC Total	0.051	0.046	0.038	0.034	0.040	0.428	0.441	0.372	0.418	0.397
BC Total	0.543	0.444	0.396	0.455	0.442	0.434	0.442	0.386	0.437	0.405
<b>Person Yrs</b>										
Direct	6.539	1.262	5.175	4.193	3.867	4.159	1.262	5.175	4.052	3.359
QCB Total	10.092	3.756	7.486	8.405	7.525	0.116	0.009	0.155	0.389	0.168
RoBC Total	1.166	1.021	0.811	0.743	0.861	8.866	4.858	9.160	8.416	7.648
BC Total	11.257	4.777	8.298	9.148	8.386	8.982	4.866	9.315	8.805	7.816
<b>Indirect Taxes</b>										
Total	32.146	75.825	44.717	31.703	31.011	34.128	76.420	37.261	27.876	32.320
Federal	13.346	31.480	18.565	13.162	12.875	14.169	31.726	15.469	11.573	13.418
Provincial	18.800	44.346	26.152	18.541	18.136	19.959	44.693	21.792	16.303	18.902
<b>Direct Taxes</b>										
Total	134.589	90.254	85.377	129.832	120.602	121.65	88.578	84.705	130.816	116.407
Federal	85.129	56.976	53.820	81.907	76.259	76.701	55.920	53.380	82.436	73.502
Provincial	49.459	33.278	31.557	47.925	44.343	44.951	32.658	31.325	48.380	42.905

in each sector in both the QCB economy and the rest of the province.<sup>4</sup> Direct impacts identify the immediate change in the sector in the region in which the unit change in output occurs; total impacts include direct effects plus the indirect effects that follow through subsequent rounds of spending. If multipliers relating

4. Government revenues are defined in gross terms excluding offset impacts on government transfer expenditures that would result from changes in household income and employment.

total to direct effects on GDP, household income, employment or government revenues are of interest, they can be readily computed from the table.

It is to be noted that impact results are simulated values that cannot be specified with confidence intervals and are contingent upon the robustness of a SAM approach to modeling as well as the data used to construct the SAM. One option in practice is to apply sensitivity analysis to the ratios by attaching chosen bounds around their point values.

The variation in impact across industries, regions and measures of impact reflects differences in production functions and tax rates across industries as well as differences in industrial mix and trading relations across regions. Salient observations are as follows. First, the variation in impact across industries and regions (irrespective of measure of impact) confirms the value of using industry-specific regional multipliers rather than generic provincial or regional multipliers. Second, it is not just that the absolute size of the impact of an exogenous change by industry varies across regions and measures of impact but that the relative size of industrial impact ratios varies by region of exogenous shock and measure of impact. For example, the largest impact on the province as a whole in terms of household income and employment from an exogenous change in the QCB region is in forestry and logging. For an exogenous change in the rest of BC, by contrast, the largest impact in terms of household income is in fishing, hunting and trapping, and in terms of employment it is in seafood products.

Third, for the sectors shown and on every measure, the secondary impact in the other region of an exogenous change in output is greater if the exogenous change occurs in the QCB region than if it occurs in the rest of the province. This is because input spending leakages to the other region within the province are greater from the smaller QCB region than from the rest of the province. A corollary is that the proportion of total BC impact retained in the QCB region if the initial change in output occurs in that region is lower than the proportion retained in the rest of the province if the change in output occurs in the rest of the province. It is notable, however, that the proportion of total impact retained in the QCB region if the initial change in output occurs in the QCB region is relatively high in the key industries of the QCB – at least 89%, except in the case of jobs (person years of employment) in fishing, hunting and trapping where the proportion is still 79%.

## Conclusions

The prototype model outlined here illustrates the feasibility of developing SAMs for more than one region at the sub-provincial level using locally specific data in combination with provincial and national data.

The advantage of the model is that it facilitates simultaneous simulations at regional and provincial levels taking into account feedback effects between regions. The impact ratios generated from the model provide simulated values for the impact of policy or other exogenous changes in the economic environment on regional and provincial output, GDP, household income and employment, as well as on revenues of different levels of government. Also, the combined provin-

cial/regional SAM is to be used as the database for calibration of a bi-regional CGE model.

For sectors shown, results confirm the value of using industry-specific regional multipliers rather than generic provincial or regional multipliers. Also, results show that the relative as well as the absolute sizes of impact of an exogenous change in output by industry vary according to region and measure of impact. Results further indicate that exogenous changes have greater impacts in the other region if they occur in the QCB region than if they occur elsewhere in the province. Similarly, the proportion of non-tax impact retained in the region in which exogenous changes occur is lower in the QCB region than in the rest of the province. Even so, at least 89% of the impact is typically retained in the QCB region.

The model remains a work in progress. The plan is to extend the model by disaggregating households by income and/or ethnic class and to include induced as well as indirect impacts in the results generated from the model. Impact ratios for other sectors not currently formulated in the BCIOM, notably tourism, could be approximated from the SAMs using additional extraneous information. Also, further refinement of assumptions and data may be possible over time. Moreover, the model will be continuously updated as data for more recent years become available.

Of course, the limitations of the SAM are to be kept in mind. Like IO models, SAMs remain fixed price/ fixed coefficient models that assume no supply constraints and are estimated on the basis of several key assumptions due to lack of exact data. Impact results are therefore only as good as the model and data used for generating them. However, economic modeling is rarely, if ever, a counsel of perfection, and despite limitations, the bi-regional sub-provincial SAM offers an advance on simple economic base models as well as on more sophisticated general equilibrium approaches that rely at the regional level exclusively on transactions data estimated for other jurisdictions, provide impact estimates for a single region without taking into account interregional feedback effects in the spending stream, do not disaggregate institutions and/or focus only at the provincial level.

## References

- Alavalapati, J., B. White, P. Jagger and A. Wellstead. 1996. "Effect of Land Use Restrictions on the Economy of Alberta: A Computable General Equilibrium Analysis". *Canadian Journal of Regional Science*, 19: 349-365.
- BC Stats. Undated. *Regional and Community Facts*. (Census 2001), <http://www.bcstats.gov.bc.ca/data/dd/facsheet/facsheet.htm>.
- \_\_\_\_\_. 2001. *BC Manufacturing Statistics by Regional District, 1999*. Victoria: BC Stats.
- Binkley, C.S., M. Percy, W. Thompson and I. Vertinsky. 1994. "A General Equilibrium Analysis of the Economic Impact of a Reduction in Harvest Levels in British Columbia". *The Forestry Chronicle*, 70: 449-454.
- Conference Board of Canada. 2005. <http://www.conferenceboard.ca/ctri/impact.htm>.

- Constantino, L. and M. Percy. 1988. "A Policy Simulator for the Forest Sector of British Columbia". Working Paper 109. Edmonton: University of Alberta.
- Davis, H. 1976. "An Interindustry Study of the Metropolitan Vancouver Economy". Report no. 6. Vancouver: Urban Land Economics, University of British Columbia.
- \_\_\_\_\_. 1986. "Income and Employment Multipliers for Seven British Columbia Regions". *Canadian Journal of Regional Science*, 9: 103-115.
- Decaluwé, B., A. Lemelin, V. Robichaud, D. Bahan and D. Florea. 2004. "Le Modèle d'Équilibre Général Calculable du Ministère des Finances, de l'Économie et de la Recherche du Québec : un modèle Bi-Régional du Québec et du Reste-du-Canada", dans L. Cloutier et C. Debresson, avec la collaboration de É. Dietzenbacher. *Changement Climatique, Flux Technologiques, Financiers et Commerciaux – Nouvelles Directions d'Analyse Entrée-Sortie*. Actes de la Quatorzième Conférence Internationale de Techniques d'Analyse Entrée-Sortie, October 10-15 2002, Montréal: Presses de l'Université du Québec, 285-297.
- Decaluwé, B., A. Lemelin, V. Robichaud and D. Bahan. 2003. "General Equilibrium Model of the Ministère des Finances du Québec (GEMFQ): Characteristics and Structure of the Model". [http://www.finances.gouv.qc.ca/en/ministere/bourses/pdf/2003\\_002\\_eng.pdf](http://www.finances.gouv.qc.ca/en/ministere/bourses/pdf/2003_002_eng.pdf).
- Duhaime, G. 1991. "Contraintes économiques à l'autonomie: une matrice de comptabilité sociale du Nunavik (Canada)". *Canadian Journal of Regional Science*, 14: 93-112.
- Fréchette, P., P. Villeneuve, M. Boisvert and M. Thériault. 1992. "L'évaluation des retombées économiques régionales de l'Université Laval à l'aide d'un modèle calculable d'équilibre général". *Revue Canadienne des Sciences Régionales*, 15:81-100.
- Government of British Columbia. 1999. *The British Columbia Macroeconomic Model*. Victoria, BC: Ministry of Finance and Corporate Relations.
- \_\_\_\_\_. Undated. *Economic Impact Assessment using the BC Input-Output Model*. Victoria, BC: Treasury Board Secretariat, Ministry of Finance and Corporate Relations.
- Horne, G. 2004. *British Columbia's Heartland at the Dawn of the 21<sup>st</sup> Century: 2001 Economic Dependencies and Impact Ratios for 63 Local Areas*. Victoria, BC: BC Ministry of Supply.
- Lemelin, A. 1993. "Note: Matrices régionales de comptabilité sociale et flux implicites d'échanges inter-régionaux". *Revue Canadienne des Sciences Régionales*, 15: 121.
- \_\_\_\_\_. 1994. "Analyse économique régionale et équilibre général : un modèle appliqué à la Région Métropolitaine de Montréal". *Revue d'Économie Régionale et Urbaine*, 5 : 795-820.
- \_\_\_\_\_. 1998. "The Impact of an Identical Demand Shock on Two Cities". *Growth and Change*, 29: 215-229.
- Minnesota IMPLAN Group Inc. 1999. *Users Guide, IMPLAN Professional Version 2.0*. <http://www.implan.com>, Stillwater, Minnesota.

- National Institute of Economic and Industrial Research (NIEIR). Undated. *YourPlace-IO*, <http://www.nieir.com.au/code/yourplace/>, Melbourne, Australia.
- Patriquin, M., J. Alavalapati, A. Wellstead and W. White. 2002. "A Comparison of Impact Measures from Hybrid and Synthetic Techniques: A Case Study of the Foothills Model Forest". *Annals of Regional Science*, 36: 265-278.
- Patriquin, M., J. Alavalapati, A. Wellstead, S. Young, W. Adamowicz and W. White. 2003. "Estimating Impacts of Resource Management Policies in the Foothills Model Forest". *Canadian Journal of Forest Research*, 33: 147-155.
- Percy, M. and L. Constantino. 1989. *The Softwood Lumber Dispute and its Impact on the Economy of British Columbia: a General Equilibrium Analysis*. Working Paper 125. Vancouver, BC: Forest Economics and Policy Analysis Research Unit, University of British Columbia.
- Price Waterhouse. 2000. *The Forest Industry in British Columbia, 1999 Report Tables*. Vancouver, BC.
- Sandhu, G.S. 2000. *BC Forest Policy Analysis: an Application of a Computable General Equilibrium Model*. Victoria, BC: Rreport for BC Ministry of Forests.
- Sandhu, G. S. and M. Percy. 1997. *Computable General Equilibrium Model of the Alberta Economy*. Edmonton, AB: Report for Alberta Agriculture, Food and Rural Development.
- Siemens, J.K. and S.N. Kulshreshtha. 1995. *An Economic Impact Assessment Model for the Prince Albert Model Forest Region*. Saskatoon, Sask: Report for Prince Albert Model Forest Association, Department of Agricultural Economics, University of Saskatchewan.
- Statistics Canada. 1999. *Annual Survey of Manufactures, Principal Statistics 1999, catalogue no. 31-203*. Ottawa, Ontario: Queen's Printer.
- \_\_\_\_\_. 2002. *Provincial Economic Accounts: Annual Estimates, catalogue no.13-213*. Ottawa, Ontario: Queen's Printer.
- \_\_\_\_\_. 2003a. *Input Output Accounts*, CANSIM tables 381-009, 381-012, 381-0013, 386-0002 and 383-0009.
- \_\_\_\_\_. 2003b. *Provincial Gross Domestic Product by Industry, 1997-2002*, CANSIM Table 379-0025
- Synergy Management Group Ltd. 2004. *Analysis of Regional Employment*. Victoria, BC: Series of reports for BC Ministry of Sustainable Resource Management.







## Appendix B

### An Algebraic Description of the Model

Notations used are as follows:

$L$	=	QCB region;
$M$	=	RoBC region;
$n_i$	=	number of industries in a region. It is assumed that both regions L and M have the same number of industries and the same number of commodities;
$Mcw_i^R$	=	imports of commodity $i$ from rest of Canada and world to region $R = L$ or $M$ . (not including imports from the other region of BC);
$Xcw_i^R$	=	exports of commodity $i$ to rest of Canada and world from region $R = L$ or $M$ . (not including exports to the other region of BC);
$n_c$	=	number of commodities (goods and services);

$q = \begin{pmatrix} q^L \\ q^M \end{pmatrix}$  is a vector of domestically produced commodities and  $q^R = (q_j^R)$ ,  $q_j^R$  represents the value of the  $j^{\text{th}}$  commodity produced in the region  $R = L$  or  $M$  and  $j = 1, 2, \dots, n_c$ ;

$g = \begin{pmatrix} g^L \\ g^M \end{pmatrix}$  and  $g^R = (g_i^R)$ ,  $g_i^R$  is the value of the  $i^{\text{th}}$  industry outputs (or total inputs) in region  $R = L$  or  $M$  and  $i = 1, 2, \dots, n_c$ ;

$U = \begin{pmatrix} U^{LL} & U^{LM} \\ U^{ML} & U^{MM} \end{pmatrix}$  is the use matrix for BC,  $U^{LL} = \mathbf{u}_{ij}^{LL}$ , is a matrix of order  $(n_c \times n_c)$  whose elements are defined as  $\mathbf{u}_{ij}^{LL}$ , the value of the  $i^{\text{th}}$  commodity produced in region L and used as an input by the  $j^{\text{th}}$  industry in region L. Furthermore,  $\mathbf{u}_{ij}^{LM}$  represents the value of the  $i^{\text{th}}$  commodity produced in region L and used as input by the  $j^{\text{th}}$  industry in region M. The rest of the components of U are defined similarly;

$$B = \begin{pmatrix} B^{LL} & B^{LM} \\ B^{ML} & B^{MM} \end{pmatrix} \quad \text{is the technical coefficients matrix for BC. } B^{LL} = \mathbf{b}_{ij}^{LL}, \quad \mathbf{b}_{ij}^{LL} = \mathbf{u}_{ij}^{LL} / g_j. \text{ The other components of B are defined similarly;}$$

$$V = \begin{pmatrix} V^L & \phi \\ \phi & V^M \end{pmatrix} \quad \text{is the BC make matrix. } V^R = \mathbf{v}_{ij}^R, \text{ an } (n_i \times n_c) \text{ make matrix for region } R \text{ whose elements } v_{ij} \text{ represents the value of the } j^{\text{th}} \text{ commodity output produced by the } i^{\text{th}} \text{ industry in the region } R = L \text{ or } M, \text{ and } \phi \text{ is a null matrix;}$$

$$D = \begin{pmatrix} D^L & \phi \\ \phi & D^M \end{pmatrix} \quad \text{the domestic market share matrix of BC and } D^L = \mathbf{d}_{ij}^L, \quad \mathbf{d}_{ij}^L = \mathbf{v}_{ij}^L / q_j^L, \quad \phi \text{ is an empty matrix and } D^M \text{ is defined similarly;}$$

$$\begin{aligned} e &= \text{domestic final demand (the sum of personal expenditures, gross fixed capital formation, government current spending on commodities and additions to inventories); and} \\ v w_j^R &= \text{value of inventory withdrawals of the } j^{\text{th}} \text{ commodity } (j = 1, 2, \dots, n_c) \text{ in region } R = L \text{ or } M. \end{aligned}$$

Using the identity that the commodity supply equals the commodity demanded in each region, we have

$$\begin{aligned} q_i^L + M c w_i^L + v w_i^L = \\ (\mathbf{u}_{i1}^{LL} + \mathbf{u}_{i2}^{LL} + \dots + \mathbf{u}_{in_i}^{LL}) + (\mathbf{u}_{i1}^{LM} + \mathbf{u}_{i2}^{LM} + \dots + \mathbf{u}_{in_i}^{LM}) + \\ e_i^L + X c w_i^L \end{aligned}$$

$$\begin{aligned} q_i^M + M c w_i^M + v w_i^M = \\ (\mathbf{u}_{i1}^{ML} + \mathbf{u}_{i2}^{ML} + \dots + \mathbf{u}_{in_i}^{ML}) + (\mathbf{u}_{i1}^{MM} + \mathbf{u}_{i2}^{MM} + \dots + \mathbf{u}_{in_i}^{MM}) + \\ e_i^M + X c w_i^M \end{aligned}$$

for  $i = 1, 2, \dots, n_c$

Or,

$$q = \begin{pmatrix} U^{LL} & U^{LM} \\ U^{ML} & U^{MM} \end{pmatrix} + \begin{pmatrix} e^L \\ e^M \end{pmatrix} + \begin{pmatrix} Xcw^L \\ Xcw^M \end{pmatrix} - \begin{pmatrix} Mcw^L \\ Mcw^M \end{pmatrix} - \begin{pmatrix} vw^L \\ vw^M \end{pmatrix}$$

or

$$q = \begin{pmatrix} B^{LL} & B^{LM} \\ B^{ML} & M^{MM} \end{pmatrix} + \begin{pmatrix} g^L \\ g^M \end{pmatrix} + e + Xcw - Mcw - vw$$

or

$$q = Bg + e + Xcw - Mcw - vw$$

or

$$Dq = DBg + De + DXcw - DMcw - Dvw \quad (B1)$$

Since,  $\sum_{j=1}^{n_c} v_{ij}^R = g_i^R$ , implies that  $D^R q^R = g^R$  ( $R = L$  or  $M$ ). Substituting this relation in equation (B1) we have,

$$q = DBg + De + DXcw - DMcw - Dvw \quad (B2)$$

The import coefficients ( $u_j^R$ ) for rest of Canada and the world for commodity  $j = 1, 2, \dots, n_c$  and  $R = L$  or  $M$  are calculated using the following relationships

$$Mcw_i^L = u_i^L \left( \sum_{j=1}^{n_i} (b_{ij}^{LL} g_j^L + b_{ij}^{ML} g_j^M) + e_i^L \right)$$

$$Mcw_i^M = u_i^M \left( \sum_{j=1}^{n_i} (b_{ij}^{ML} g_j^L + b_{ij}^{MM} g_j^M) + e_i^M \right)$$

The coefficient of the inventory withdrawals ( $\lambda_j^R$ ) for commodity  $j = 1, 2, \dots, n_c$  and  $R = L$  or  $M$  are calculated using the following relationships

$$vw_i^L = \lambda_i^L \left( \sum_{j=1}^{n_i} (b_{ij}^{LL} g_j^L + b_{ij}^{ML} g_j^M) + e_i^L + Xcw_i^L - Mcw_i^L \right)$$

$$vw_i^M = \lambda_i^M \left( \sum_{j=1}^{n_i} (b_{ij}^{ML} g_j^L + b_{ij}^{MM} g_j^M) + e_i^M + Xcw_i^M - Mcw_i^M \right)$$

Substituting these coefficients in equation (B2) for  $Mcw_i$  and  $vw_i$ , the reduced form equation can be written as

$$g = [I_1 - D(I_2 - \lambda)(I_2 - u)B]^{-1} D(I_2 - \lambda)[(I_2 - u)e + Xcw] \quad (B3)$$

Where  $I_1$  is an identity matrix of an order  $(2n_i \times 2n_i)$  and  $I_2$  is an identity matrix of order  $(2n_c \times 2n_c)$ ,  $\lambda$  and  $u$  are diagonal matrices of order  $(2n_c \times 2n_c)$  with diagonal elements  $\{u_1^L, u_2^L, \dots, u_n^L, u_1^M, u_2^M, \dots, u_n^M\}$  and  $\{\lambda_1^L, \lambda_2^L, \dots, \lambda_{n_c}^L, \lambda_1^M, \lambda_2^M, \dots, \lambda_{n_c}^M\}$ .

Total impacts (direct and indirect) of exogenous shocks in QCB (L) and/or RoBC (M) regions are calculated using Equation (B3). The matrix  $[I_1 - D(I_2 - \lambda)(I_2 - u)B]^{-1}$  is the total requirements matrix which can also be called the Leontief Inverse for a bi-regional model.