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QUALITATIVE MULTICRITERIA EVALUATION METHODS FOR DEVELOPMENT PLANNING

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Introduction

In the past decade there has been a growing interest in evaluation research. Much of the impetus for doing evaluation studies in urban and regional planning has come from the recognition that planning involves choices. The purpose of evaluation research is to help (or sometimes influence) administrators to make better decisions in a particular situation than would otherwise have been made.

Evaluation research is normally viewed as a procedure for testing the effectiveness of particular alternative choice possibilities (e.g., plans, programs, courses of action, strategies). Such a perspective, however, assumes the existence of evaluable alternatives which meet a number of preconditions: a) precisely defined alternatives; b) clearly specified evaluation criteria; and c) an explicit insight into existing value structures. In the past attention has been focused mostly on evaluation methods which assume that these preconditions can be fulfilled. In addition to conventional *monetary* evaluation methods, such as social cost-benefit analysis and cost-effectiveness analysis, several *non-monetary* methods have been developed which are based on the multidimensionality of the various evaluation criteria (see, among others, Roy [11], Van Delft and Nijkamp [13], Voogd [15]). Yet it is rare to find evaluation problems in urban and regional planning practice which adequately meet these preconditions.

Evaluation research in urban and regional planning is often hampered by the lack of reliable *quantitative* information. Many variables and attributes are only measureable or measured on an ordinal or *qualitative* scale. Recently some interesting qualitative evaluation methods have been developed, which are capable of taking into account ordinal information. This paper is devoted to a discussion of these new evaluation methods.

Qualitative multicriteria evaluation methods can be distinguished into three categories: a) ratio-scale methods; b) frequency methods; and c) scaling models. Each category will be examined in this paper and some recently developed evaluation methods presented. The various multicriteria methods will be illustrated by means of an empirical application to an important regional planning problem in The Netherlands. This concerns the evaluation of several development strategies for the urbanized western part of the country.

The Structure of a Multicriteria Evaluation Procedure

Suppose we want to evaluate a finite set of alternatives i ($i = 1,2,\ldots,I$) by means of a finite set of criteria j ($j = 1,2,\ldots,J$). It is assumed that for each criterion the alternatives can be assigned a value on an ordinal scale. Then r_{ji} represents the ranking of alternative i with respect to criterion j. It will be assumed that the "higher" r_{ji} is, the "better" alternative i is with respect to criterion j. The elements r_{ji} can be integrated into an *evaluation matrix* $\underline{R}$ (of order J x I) with the following structure:

$$\underline{R} = \begin{array}{c|ccccccc} & 1 & 2 & 3 & \cdot\cdot & I & \cdot\cdot & I \\ \hline 1 & & & & & & & \\ 2 & & & & & \cdot & & \\ \cdot & & & & & \cdot & & \\ \cdot & & & & & \cdot & & \\ j & \cdot & \cdot & \cdot & & r_{ji} & & \\ \cdot & & & & & & & \\ J & & & & & & & \end{array} \qquad (1)$$

In order to draw straightforward conclusions from (1) we need further information about the relative priorities of the various criteria. It appears from empirical research that it is not recommended that sophisticated psychological measurement techniques be used to assess preferences of decision-makers [18]. A much easier and therefore more appropriate approach is to define alternative priority sets according to a certain (political) vision (e.g., environmental vision, economic vision, social vision). Such value statements with regard to vision n ($n = 1,2,\ldots,N$) and criterion j are assumed to be represented by ordinal weights w_{nj}. These can be included in a *priority matrix* $\underline{W}$ (of order N x J) with the following structure:

$$\underline{W} = \begin{array}{c|cccccc} & 1 & 2 & \cdot & \cdot & j & \cdot\ J \\ \hline 1 & & & & & & \\ 2 & & & & & \cdot & \\ 3 & & & & & \cdot & \\ \cdot & & & & & \cdot & \\ \cdot & & & & & \cdot & \\ n & \cdot & \cdot & \cdot & & w_{nj} & \\ \cdot & & & & & & \\ N & & & & & & \end{array} \qquad (2)$$

There are various techniques available to combine the information from (1) with (2) in order to arrive at a judgement of the alternatives under consideration. This can be represented in the following way:

$$\underline{S} = \underline{R} \oplus \underline{W} \qquad (3)$$

where $\oplus$ denotes a combination in one way or another. The resulting matrix $\underline{S}$ is of order N x I. Its elements s_{ni} represent the rational preferability of alternative i with respect to vision n.

In the following sections several techniques will be discussed which can be used to operationalize formula (3). It should be noted that in each case assumptions have to be made in order to make further use of the qualitative information of (1) and (2).

Ratio-scale Methods

A great deal of sophistication has been reached in cardinal multicriteria evaluation, where both the evaluation matrix and the criterion weights are assumed to contain metric elements. Well known examples are the weighted summation technique (e.g., Hill [4]) and concordance analysis (e.g., Roy [11]; and Van Delft and Nijkamp [13]). One possible way to treat the qualitative elements of $\underline{R}$ and $\underline{W}$ is to assume that the rankings can be considered as measurements on a ratio scale. In other words, the qualitative characteristics of the data are neglected. This makes it possible to apply methods which utilize metric properties of imputed information. In this section two new ratio-scale methods will be discussed: a *generalized concordance analysis* and a *rescoring method*. For simplicity we will drop the index n in the various formulae.

In the generalized concordance analysis the alternative choice possibilities are compared pairwise to obtain information about the relationships between them. For determining the degree of dominance of alternative i above i' a concordance measure $c_{ii'}$ is used and the degree of dominance of alternative i' above i is reflected by a discordance measure $d_{ii'}$. These measures are defined in the following way:

$$c_{ii'} = \left[\frac{\sum_{j \in C_{ii'}} w_j^{\alpha}}{\sum_j w_j^{\alpha}} \right]^{1/\alpha} \qquad (4)$$

and

$$d_{ii'} = \left[\frac{\sum_{j \in D_{ii'}} \{w_j(r_{ji} - r_{ji'})\}^{\alpha}}{\sum_j \{w_j(r_{ji} - r_{ji'})\}^{\alpha}} \right]^{1/\alpha} \qquad (5)$$

where

$$C_{ii'} = \{ j \mid r_{ji} \geq r_{ji'} \} \qquad (6)$$

and

$$D_{ii'} = \{ j \mid r_{ji} < r_{ji'} \} \qquad (7)$$

By means of the *scaling parameter* $\alpha (\alpha > 0)$ the researcher is able to vary the importance of the small weights and small divergences between the rankings. A final appraisal of the alternatives is then based on a comparison of the $c_{ii'}$ and $d_{ii'}$ measures. This can be done in several ways (see, for instance, Van Delft and Nijkamp [13] and Voogd [15]). A very simple way is to use variable threshold values. The best alternatives are those that satisfy the following conditions:

$$c_{ii'} > \hat{c} \qquad (8)$$

$$d_{ii'} < \hat{d} \qquad (9)$$

By relaxing these conditions by means of the definition of lower (higher) thresholds $\hat{c}$ and $\hat{d}$ a statement can be made about the preferability of the various alternatives.

The rescoring method also determines firstly the degree of dominance of alternative i above alternative i'. This is done by two measures: formula (4) and

$$d_{ii'} = \left[\frac{\sum_{j \in C_{ii'}} \{w_j(r_{ji} - r_{ji'})\}^{\alpha}}{\sum_j \{w_j(r_{ji} - r_{ji'})\}^{\alpha}} \right]^{1/\alpha} \qquad (10)$$

where the definition of $C_{ii'}$ corresponds to (6). The next step is to define an auxiliary matrix $\underline{Z}$ of order I x I with elements $z_{ii'}$ where:

$$z_{ii'} \begin{cases} = 2 \text{ if } c_{ii'} > c_{i'i} \text{ and } d_{ii'} > d_{i'i} \\ = 1 \text{ if } c_{ii'} \geq c_{i'i} \text{ and } d_{ii'} \leq d_{i'i} \text{ or} \\ \quad\; c_{ii'} \leq c_{i'i} \text{ and } d_{ii'} \geq d_{i'i} \\ = 0 \text{ if } c_{ii'} < c_{i'i} \text{ and } d_{ii'} < d_{i'i} \end{cases} \qquad (11)$$

It can easily be shown that the row totals of matrix Z embody some unfairness [15]. To counteract these unfairnesses, Kendall [5] proposed a method to calculate weighted row sums in a symmetric matrix. This is an iterative procedure which can be described by the following formula:

$$\underline{r}_x = \underline{Z} \cdot \underline{r}_{x-1} \quad \text{(for each } x = 2,3,4,\ldots.) \qquad (12)$$

where $\underline{r}$ is a column vector with I elements representing the weights of the various alternatives (not to be confused with the priorities of (2)). In the first iteration (x = 1) all the elements of r have a value of 1. The vectors $\underline{r}_x$ have for all x only positive elements. The definite ranking of the alternatives has been reached if, after a certain x, the ranking of the alternatives on the basis of the size of the elements of $\underline{r}_x, \underline{r}_{x+1}, \ldots\ldots$ does not change any more. So the elements of vector $\underline{r}$ converge to a steady ranking, which is the preference ranking of the alternatives according to this rescoring method.

Frequency Methods

Frequency methods do not make use of (metric) dominance measures like (4) or (10) but treat qualitative information in a theoretically more elegant way, viz. by transforming it into information on a binary or nominal scale. The main assumption of frequency methods concerns the metric treatment of this transformed information, which might be debatable from a theoretical viewpoint.

Well-known frequency methods are the qualitative concordance analysis [13] and the permutation method [12]. In this section a new frequency method will be elaborated. This concerns the *numerical interpretation method.* This method is also based on a pairwise comparison of the various alternatives.

The first step is to formulate for each pair of alternatives (i, i') a J x J matrix $\underline{Q}^{ii'}$ with elements $q_{jj'}^{ii'}$ (i, i' = 1,2, . . . , I; j,j' = 1,2, . . . ,J) where:

$$q_{jj'}^{ii'} \begin{cases} = 1 & \text{if } r_{ji}>r_{ji'} \text{ and } w_j>w_{ji'} \\ = 0 & \text{if } r_{ji} = r_{ji''} \text{ and } r_{j'i} = r_{j'i'} \text{ or} \\ & \text{if } r_{ji}>r_{ji''} \text{ and } r_{j'i}<r_{j'i'} \text{ and } w_j = w_{j'} \text{ or} \\ & \text{if } r_{ji}<r_{ji'} \text{ and } r_{j'i}>r_{j'i'} \text{ and } w_j = w_{j'} \\ = -1 & \text{in all other cases} \end{cases} \qquad (13)$$

The information of the matrices $\underline{Q}^{ii'}$ can be integrated into a new matrix $\underline{M}$ of order I x I with elements $m_{ii'}$, where:

$$m_{ii} = \sum_{j} \sum_{j'} q_{jj'}^{ii'} \quad (j'>j) \qquad (14)$$

The preference score s_i for alternative i can now be defined as:

$$s_i = \sum_{i'=1}^{I} m_{ii'} \qquad (15)$$

Obviously, alternative i is more favourable as s_i is higher.

Scaling Models

A new way to perform qualitative multicriteria evaluations is by means of scaling models. Recently a new evaluation procedure has been developed, based on the principles of multidimensional scaling, by which qualitative evaluation rankings and criterion weights can be treated in a theoretically consistent way, without violating the ordinal characteristics of the imputed data (i.e., (1) and (2)). This procedure is called *ordinal geometric evaluation* [8; 9; 14; 16; 17]. The main assumption of this approach concerns the definition of the "model" itself, i.e., the definition of the geometric space in which the alternatives and criteria are scaled.

An ordinal geometric evaluation procedure consists of two stages. In the first stage two metric (cardinal) matrices $\underline{X}$ and $\underline{Y}$ are extracted from the ordinal evaluation matrix $\underline{R}$ which implies a representation of the alternatives and the ideal points of the criteria as points in a multidimensional space of a few dimensions. Matrix $\underline{X}$ is of order I x P and contains the coordinates x_{ip} of alternative i and dimension p (p = 1,2, . . . ,P). Matrix $\underline{Y}$ is of order J x P and contains the coordinates of the ideal points y_{jp} of the criteria. An ideal point can be interpreted as the "ideal value" of a criterion [2; 17].

It is easy to see that more ordinal conditions are available via the evaluation scores of matrix $\underline{R}$ than geometric coordinates x_{ip} and y_{jp} are necessary. Because such abundant information involves many degrees of freedom, it is possible to transfer these ordinal e_{ji}-scores into metric x_{ip} and y_{jp}-scores. This concerns a minimization problem, which can be formally denoted, for a geometric space with a fixed number of dimensions P, as:

$$\min_{\underline{X},\underline{Y}} \phi = f(\underline{D} - \underline{\hat{D}}) \qquad (16)$$

subject to:

$$\hat{D} \overset{m}{=} R$$

$$\underline{D} = g(\underline{Y},\underline{X})$$

where: $\underline{D}$ = rectangular matrix (of order J x I) of (unknown) geometric distances d_{ji} between criterion point j and alternative i

$\underline{\hat{D}}$ = rectangular matrix (of order J x I) of (unknown) order - isomorph values $\hat{d}_{ji}$, which have the same ranking as the imputed rankings r_{ji} (denoted by the monotonicity symbol $\overset{m}{=}$)

$g(\underline{Y},\underline{X})$ = geometric distance function.

This method is aimed at a representation of the alternatives and criterion points in a geometric space of minimum dimensionality; i.e., P should be kept as small as possible. The auxiliary matrix $\underline{\hat{D}}$ is used to do arithmetic operations with the qualitative information of matrix $\underline{R}$. Matrix $\underline{\hat{D}}$ can be determined by means of a monotone regression procedure [5; 6] or a rank image procedure [3].

By means of (16) we are able to find optimal coordinate values $\underline{X}$ and $\underline{Y}$. To evaluate the taxonomy of alternatives, embodied by matrix $\underline{X}$, we need a point of reference; i.e., an "ideal choice possibility". The coordinates of this ideal choice possibility $({}^{i}\gamma_p)$ can be found by the following formula:

$$\gamma_p = \sum_{j=1}^{J} w_j \cdot y_{jp} \qquad (17)$$

where w_j represents the metric criterion weight. It is assumed that these weights add up to unity; i.e., $\sum_j w_j = 1$.
The preference score for alternative i can now be calculated by means of a Minkowski distance metric:

$$s_i = \{ \sum_{p=1}^{P} (|x_{ip} - \gamma_p|)^c \}^{1/c} \qquad (c \geq 1) \qquad (18)$$

Any value of $c \geq 1$ may be chosen, provided that the same distance metric is used in model (16).

In (17) it is assumed that the criterion weights w_j have metric properties. If we have only ordinal priority rankings w_j then the conclusion is obvious that there is not sufficient information for a precise calculation of the coordinates of the ideal choice possibility. Hence, the only way left is to consider the area in the geometric space in which this reference point could be situated. This area is defined by the extreme values of the weights, which are in accordance with the rankings w_j. Suppose we have three criteria for which the following priorities hold: $w_1 \geq w_2 \geq w_3$. The following extreme metric weight sets can now be distinguished: (1,0,0), (1/2, 1/2, 0) and (1/3, 1/3, 1/3). For each extreme weight vector we can determine now the coordinates γ_p (formula (17)). A combined interpretation of the results of the various extreme weights enables us then to come to a final judgement with respect to the various alternative choice possibilities. This approach is empirically illustrated in Nijkamp & Voogd [9]. Evidently, the interpretation of the various preference scores resulting from the extreme weights will become more difficult if the number of criterion rankings increases. In such cases a stochastic treatment of the ideal choice possibility might be more attractive. This is elaborated in Voogd [17].

An Illustration

The various qualitative multicriteria evaluation methods will

now be illustrated by means of an application to a number of alternative development strategies for the western part of The Netherlands. In the last century a pattern of urban growth developed in the Western Netherlands which created an urban ring surrounding a rural area, which is called the Randstad (see Figure 1). This development was seen as having great value in that it allowed the urbanized area, as it grew, to spread around the periphery without having an impact on the rural countryside of the centre (i.e., the green heart), which could thus retain its original character and be used for the recreation needs of the surrounding cities.

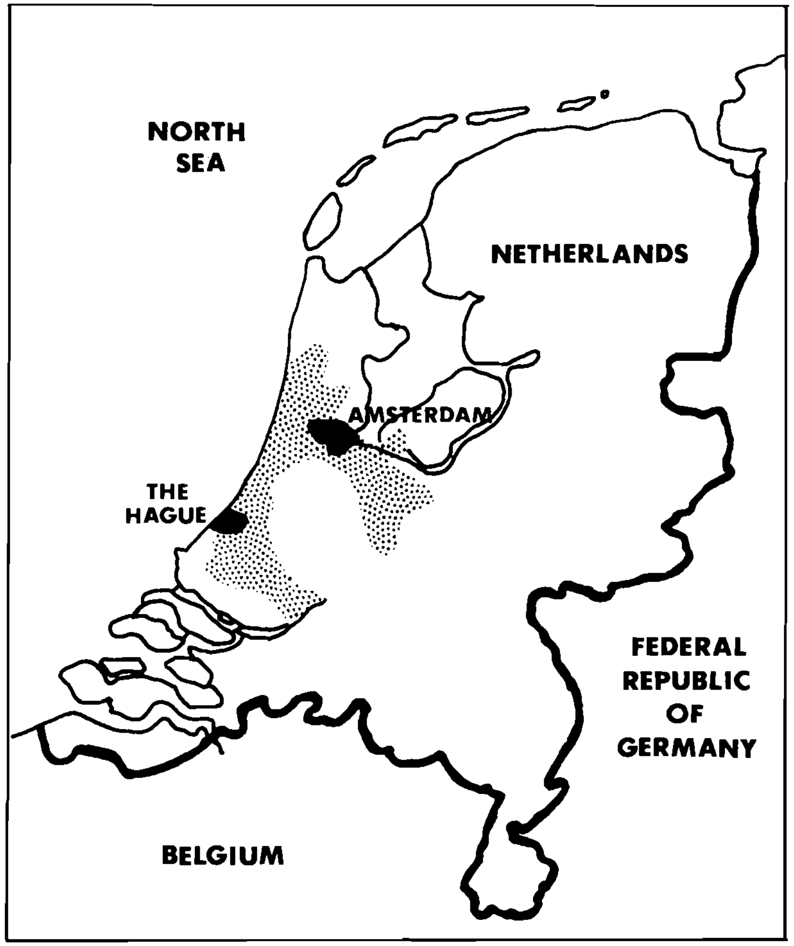

Figure 1
THE RANDSTAD AREA

This pattern of development was adopted as an official policy. The main target was to protect the green centre by concentrating growth in the surrounding periphery and in other parts of The Netherlands. However, recent growth patterns no longer follow this expected path. The outer urban ring of the Randstad has been experiencing slow growth or decline while the green centre has been experiencing a rapid expansion of urban activity. The concern of public authorities over this situation resulted in a critical examination of future development strategies [1; 10].

In this application five alternative development patterns will be considered:

I. *Unconstrained Trend*
This strategy is based on the extrapolation of present trends, modified only by those public actions which are now being implemented or are firmly committed.

II. *Constrained Trend*
This is based on the firm implementation of planning controls following from official governmental documents and consequent regional planning allocations.

III. *Outside Developments*
This strategy is designed to produce the greatest amount of urban and recreational development change and investment in areas outside the western part of The Netherlands.

IV. *Inside Developments*
This, in contrast, is designed to produce the greatest amount of urban and recreational development pressure and investment inside the green centre of the Randstad.

V. *Concentration*
This strategy is designed to create conditions likely to reverse the significant declines in activity levels and investment in the urban ring.

In this illustration of the qualitative multicriteria evaluation methods the following main criteria are taken into consideration: (A) environmental aspects, (B) agricultural aspects, (C) traffic and transportation, (D) urban facilities, and (E) economic effects. Each of these main criteria is composed of a set of subcriteria. These are elaborated in Voogd et al. [18].

For reasons of surveyability only two sets of ordinal weights are used. Each set corresponds, to a certain extent, to a future policy vision. These visions are: (1) an *economic policy* which stresses the industrial growth; (2) a *social policy* which attempts to improve the living conditions in the cities. For each policy an ordinal weight vector for the evaluation criteria is assessed. The data included in Tables 1 and 2 have been used as inputs for the methods, which have been discussed in this paper. It is assumed in both Tables that more crosses imply a better valuation.

Table 1

THE EVALUATION MATRIX

CRITERIA	ALTERNATIVES				
	I	II	III	IV	V
A	xxxxx	xx	x	xxx	xxxx
B	xx	xxxx	xxxxx	x	xxx
C	xx	xxxx	xxxxx	x	xxx
D	x	xxxx	xxxxx	xx	xxx
E	xxxxx	xx	x	xxxx	xxx

Table 2

ALTERNATIVE ORDINAL WEIGHT SETS

VISION	CRITERIA				
	A	B	C	D	E
1	xxx	xxxxx	xxxx	x	xxxx
2	xxx	xx	xxxx	x	x

The following methods have been applied: a) the generalized concordance analysis with scaling parameter $\alpha = 1$; b) the generalized concordance analysis with scaling parameter $\alpha = \infty$; c) the rescoring method; d) the numerical interpretation method; e) the ordinal geometric evaluation method. The results are given in Table 3, where a lower number means a better appraisal.

The results of the ordinal geometric evaluation method in Table 3 have been obtained by treating the ordinal rankings in Table 2 as if they were metric weights. As mentioned before, it is also possible to use the ordinal criterion rankings in a theoretically more consistent way by considering the extreme metric weights which are in agreement with the proposed priorities. In that case the ordinal geometric evaluation yields the followings results (Table 4).

Table 3

THE RESULTS OF THE QUALITATIVE MULTICRITERIA EVALUATION

METHOD	VISION 1					VISION 2				
	I	II	III	IV	V	I	II	III	IV	V
a) Generalized Concord. I	1	2	2	3	2	1	2	2	3	1
b) Generalized Concord. II	1	2	2	3	2	1	2	2	3	1
c) Rescoring Method	4	2	1	5	3	4	2	1	5	3
d) Numerical Interpretation	4	2	1	5	3	2	3	2	4	1
e) Ordinal Geometric Evaluation	1	4	5	2	3	1	3	5	4	2

Table 4

THE RESULTS OF THE ORDINAL GEOMETRIC EVALUATION FOR THE VARIOUS EXTREME WEIGHT SETS

	EXTREME METRIC WEIGHTS					PREFERENCE RANKING				
		A	B	C	D	I	II	III	IV	V
VISION 1	0	1	0	0	0	4	2	1	5	3
	0	1/2	1/2	0	0	4	2	1	5	3
	1/3	1/3	1/3	0	0	1	4	5	2	3
	1/4	1/4	1/4	0	1/4	1	4	5	3	2
	1/5	1/5	1/5	1/5	1/5	1	4	5	2	3
VISION 2	0	0	1	0	0	4	2	1	5	3
	1/2	0	1/2	0	0	1	3	4	5	2
	1/3	1/3	1/3	0	0	1	4	5	3	2
	1/5	1/5	1/5	1/5	1/5	1	4	5	2	3

Table 3 shows that the final preference rankings of the rescoring method and the numerical interpretation method differ from the outcomes of the other approaches. Evidently, these two methods stress the good position of Alternative III with respect to criteria B, C and D. For all other methods Alternative I prevails, whereas Alternative III is second-best. The ordinal geometric evaluation results in a very low preference for Alternative III if the ordinal characteristics of the weights are not taken into consideration. However, the results of Table 4 show that this bad position of Alternative III only occurs if we do not stress the differences between the criterion priorities. Table 3 does not show any difference between the two variants of the generalized concordance analysis. This is a coincidence, since other empirical research suggests that the scaling parameter can largely influence the final results [18].

In conclusion, it appears that this global analysis is in favour of a continuation of present trends in the Western Netherlands. Nevertheless, there is some evidence to suggest that a further development outside the urban ring should be pursued. For a more straightforward conclusion with regard to the various development strategies a more detailed analysis is necessary; however, this falls outside the scope of this paper.

Conclusion

This paper has focused attention on several new multicriteria methods, which can be used for a qualitative evaluation of alternative choice possibilities. It appears that the most striking aspect of the results is the lack of agreement between the methods. This can be explained by the fact that the results of each method are to some degree affected by the choice and consequences of the assumptions of the method. This evokes uncertainty, which is often neglected in evaluation research. This *"method uncertainty"* is manifest in two ways: on the one hand in the *structure* of the evaluation method (e.g., concordance analysis versus geometric evaluation), and on the other hand in the *content* of this structure (e.g., the way in which dominance functions are specified in a concordance analysis). A numerical examination by means of a Monte Carlo analysis does indicate that both components indeed influence the final outcome of an evaluation [18]. If completely independent criteria are assumed, the method uncertainty — defined as the average probability that the result of a method deviates from the result of any other method — could even exceed 40 percent. Of course, in practice this percentage will be lower due to the dependencies between criteria, but the existence of this method uncertainty reveals enough evidence to suggest as a "golden rule" that in an empirical application of a multicriteria evaluation more than one method must be used. This should not cause much practical difficulty since most evaluation methods are implemented on a computer.

In conclusion one might say that each method is based on certain specific assumptions. This implies that each method has certain arbitrary elements, which can influence the final outcomes. It is for this reason almost impossible to proclaim a method being "the best". Our recipe for treating this kind of uncertainty will undoubtedly increase the amount of output. However, if properly presented, the results of multiple methods will increase not only the amount but also the quality of the information the decision makers have on which to base their decisions.

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