

SUPPLY EFFECTS IN REGIONAL ECONOMIC MODELS

J. H. P. Paelinck and Sj. Wagenaar
Faculty of Economics
Erasmus University, Rotterdam
3000 DR Rotterdam
The Netherlands

Introduction

Since economists became concerned with economic development, especially in its structural (i.e., sectoral or regional) aspects, they have tended towards a distinction between supply and demand (push and pull) effects. The former were meant to describe (analytically) a sequence of investments, based on the presence of suppliers of one or another of the inputs needed for the new technical activity; the latter were intended to represent the emergence of a new activity (or activities) as the result of a pre-existing demand, either final or intermediate (in input-output terms).

Hirschman, for instance, in his *Strategy of Economic Development*, observes that "the lack of interdependence and linkages ... is one of the most typical characteristics of underdeveloped economies" [7:109]. Interdependence in the input-output sense is largely the result of industrialization; an insight into the most efficient sequence for industrialization of an underdeveloped area is a necessary condition for a good development policy. One way to acquire that insight is to measure the degree of interdependence of various industries in *developed* economies with the help of their input-output tables; the identification of so-called "key" or "leading" sectors is especially important. In measuring the extent to which any one industry interlocks with others, a distinction should be made between two kinds of linkage effects, called the backward effects and the forward effects, the distinction being based on the recognition of two mechanisms: (1) the input provision; that is, every non-primary economic activity will induce attempts to supply through domestic production the inputs needed in that activity; and (2) the output utilization; that is, every activity that does not by its nature cater exclusively to final demands will induce attempts to utilize its outputs as inputs in some new activities [7:100].

Input provision and backward linkage are two aspects of the same phenomenon; so are output utilization and forward linkage. The

analysis of interindustrial interdependence is performed in the economic literature with the help of indexes of backward and forward linkages;¹ these indexes are as a rule defined in relation to an open static Leontief input-output system for a national economy. This system is usually applied to problems involving the specification of final demands and determination of production levels; Oosterhaven [12:6], for example, observes that *input-output analysis* considers only backward linkages, leaving forward linkages out of account. In his approach, backward linkages relate production to demand, and are equal to the so-called indirect effects of final demand on sectoral production, while forward linkages relate supply to production, and cannot be defined within a traditional Leontief input-output model. Oosterhaven gives the following example of a forward linkage:

In the sphere of the agro-industrial complex the volume agricultural production is . . . decisive for the production volume of the foodstuff industry based on local agricultural inputs. Input-output analysis, to the contrary, assumes that a rise in demand for foodstuffs will invariably lead to a rise in the supply of the agricultural inputs required. The limited capacity and flexibility of the indispensable production factor 'land' prohibits considerable backward effects on agriculture.²

In the paper mentioned the author presents an open input-output model for a region with built-in forward linkages, developed by him and others when investigating the employment effects of a reclamation project.

No systematic view of intergrating supply effects in economic models has been presented as far as the authors know; they intend to present, within a classification scheme, such a view, based largely on their own work. The following distinction should be made:

- the supply effects of a pre-existing activity or the emergence of new activities are either presented in isolation (second section), or introduced, through specific coefficients, into an overall econometric model (third section);
- the analysis is either national (or more generally, a-spatial) or specifically regionalized (third and fourth sections); the latter idea is borrowed from location theory in which the co-ordinates of the supply sites of raw materials or intermediate products determine the optimum location of a new activity [14:Chs. 2 and 3].

¹The theoretical case in which an input-output matrix can be fully triangularized, interindustrial interdependence emerging as a hierarchy of sectors, has been left out of account here.

²Our translation.

The organization of this paper has thus clearly been shown, and we will proceed at once. One proviso: models may combine several aspects, as will already be evident from the regionalization present in the next supply-"impulse" or "shock" model.

Computing Supply Effects of Large Projects: The Case of Airports³

To determine the optimum location of a new airport, an important element to be considered is the regionalized economic impact of its activities; the results of analyzing this impact would constitute one row of the fundamental decision matrix of multicriteria analysis [11].

This section concentrates on the structure of a model which can help us to compute the economic impacts of airport activity, and in particular the supply side of these impacts; Table 1 summarizes the main elements of such a model.

Table 1

Categories of activities	DFS	DDS	P	}	R ₁ , ..., R ₁₁
		-----	---		
Effects		- DIA	P	G	
Direct	x*				X X X X X X X X
Indirect					
R ₁	X				X X
.	.				.
.	.				.
.	.				.
.	.				.
.	.				.
R ₁₁	X				X X

³For details, see [13].

The relevant activities have been classified as follows:

- DFS: Direct Functional Services, representing the *exogenous input* to the model, and taken from the airport blueprint; ground control, refuelling, luggage and freight handling, and check-in counters are only a few examples to illustrate this category.
- DDS: Derived Direct Services, linking the airport zone (symbolized as a closed set of air-ground activities) to the "rest of the world"; bussing, cabbage, trucking, phoning, and cabling are points in case. Passengers' (P) and goods' (G) activities should be considered separately; their emergence rests on a *demand effect*.
- DIA: Derived Induced Activities, being due a "supply" or "push" effect of the airport; i.e., the new technical-cum-locational facilities it offers. Examples are storing-cum-dispatching firms for fresh fruits and vegetables, firms locating near the airport owing to better (international) access to markets for inputs or outputs (lowering of the friction cost with respect to a profitability threshold).

The relevant effects should be classified as *direct* and *indirect* effects, the indirect effects being input-generated and income-generated demand effects; finally the DDS, DIA and indirect effects are to be regionalized.⁴

Formally, the model should permit the computation of an $11 \times n$ coefficient matrix, n corresponding to the number of alternative locations to be studied. The fact that a matrix and not a single vector is to be constructed is due to the *regional specificity* of the impacts, which in turn springs from the fact that alternative locations in space will produce different spatial impact patterns, as will be clarified below. The impacts, in absolute values, could then be derived from

$$X = Ax^* \quad (1)$$

where A is the $11 \times n$ coefficient matrix alluded to above, x^* the exogenous (scaler) impact (i.e., the level of DFS), and X the resulting regionalized level matrix; for alternative k ($1 \leq k \leq n$) the regionalized impacts would be

$$x_k = a_k x^* \quad (2)$$

where a_k is the k^{th} column vector taken from matrix A .

Consider now the way column a_k is built up.

⁴The regional index, r , runs from 1 through 11 (number of provinces in the Netherlands).

Start from the DFS, x^* ; they will generate a vector of multisectoral-multiregional indirect demands, d_k , of order $[(11 \times s) \times 1]$, where s is the number of sectors distinguished. Regions are ordered first, then sectors.

In the second place, x^* will generate four vectors of the same order as d_k ; call them d_{ki} $i = 1, 2, 3, 4$. These will correspond to DDS (P, G) and DIA (P, G). Applying to them a full-information intersectoral-interregional multiplier generates the total effects (direct, δ_{ki} , plus indirect) [14:Ch. 5]. Finally the (11×1) vector a_k is generated as

$$a_k = J [d_k + \sum_{i=1}^4 M_i \delta_{ki}] x^{*-1} \quad (3)$$

where M_i are the $[(11 \times s) \times (11 \times s)]$ multipliers referred to above,⁵ and J an $[11 \times (11 \times s)]$ summation matrix, built up of separate $(1 \times s)$ unit row vectors, the sectoral results per region being added up.

We now concentrate on d_k . The solution chosen to compute this vector was to start the analysis in terms of π_{il} 's, the (unknown) proportions of production values p_{il} oriented towards the airport, in the sense that their emergence and continuing existence are guaranteed only through airport facilities (externalities). The problem is to compose a table such as Table 2, for which the following assumptions are introduced:

Table 2

	1,, 11
1	π_{il}
.	
.	
.	
.	
.	
.	
.	
.	
.	
s	

A 1: distances from the airport to the regions are known;

A 2: all of the (material) outputs and inputs are shipped through

⁵In fact they are semi-input-output multipliers [9:Ch. 5].

the airport; this leads to a *sector-specific*⁶ coefficient $1 + \alpha_i$, where α_i is the proportion of total material inputs per unit output;

A 3: total volume (value) shipped through the airport on behalf of all plants is known;

A 4: the total ton (value) miles transported and shipped through the airport is known.

A "minimum-knowledge approach" can be set up using entropy maximization [19] that leads to

$$\max - \sum_{ij} \pi_{ij} \ln \pi_{ij} \quad (4)$$

with the restrictions

$$\sum_{ij} \pi_{ij} p^*_{ij} t^*_i = V = \alpha^* S \quad (5a)^7$$

$$\sum_{ij} \pi_{ij} p^*_{ij} t^*_i d^*_1 = D \quad (5b)$$

A typical solution equation is

$$-\ln \pi_{ij} - 1 - \lambda (p^*_{ij} t^*_i) - \mu (p^*_{ij} t^*_i d^*_1) = 0 \quad (7)$$

which equation, introduced into (5a) and (5b), and summed over all i and 1 , leads to a system of two equations allowing the computations of λ and μ , the Lagrange parameters [3].

The method can clearly be extended to the DIA(P) activities, which essentially relate to service sectors (hotels, travel agencies, banks).

The results for the Netherlands (1975) are shown in Table 3.

They appear rather high, especially for Zestienhoven and Beek, but other studies, having correctly computed *all long-term effects*, arrive at the same conclusion [2].

⁶Perhaps sector- and region-specific, if α_i , the proportion of total national inputs (see [14:Ch. 5]) varies per region; a correction for the 100 percent air-shipping hypothesis can be made with the help of the ratio of air vs non-air transport from the I-O table.

⁷ $t^*_i = 1 + \alpha_i$; starred symbols are exogenous (known). If sectoral volumes were available (5a) would be extended to

$$\sum_i \pi_{ij} p^*_{ij} t^*_i = V_i, \forall i \quad (6)$$

giving, of course, more precise results. We suppose, moreover, that V is a constant proportion α^* of total airport (goods) activities; later on we will indicate how α^* can be computed.

Notice that factor α^* from expression (5a) could be set at .3, from direct observation on the level of DIA in one Dutch province (North Holland). Finally, the *relative* spread of the DDS + DIA effects among Dutch provinces is given in Table 4.

Table 3
(Multipliers DDS + DIA)

Airport	Multiplier
Schiphol	2.8160
Zestienhoven	4.1035
Beek	10.4017
Eelde	2.2125

Table 4

Airports	Schiphol	Zestienhoven	Beek	Eelde
Provinces				
Groningen	.0454	.0245	.0418	.4330
Friesland	.0591	.0180	.0291	.0206
Drenthe	.0446	.0094	.0173	.4537
Overijssel	.0374	.0258	.0455	.0107
Gelderland	.0869	.0428	.0755	.0206
Utrecht	.1410	.0472	.0564	-
North Holland	.2983	.0453	.0709	-
South Holland	.1671	.6929	.1600	-
Zeeland	.0389	.0176	.0255	.0412
North Brabant	.0464	.0459	.0777	.0103
Limburg	.0349	.0302	.4018	.0099
Total	1.-	1.-	1.-	1.-

Integrating Supply Effects In Input-Output Models

As already briefly mentioned, input-output tables are the reference schemes for studying supply (push, forward) and demand (pull, backward) effects, and we will start from there, introducing some regional aspects; a more refined version of these aspects' modelling will be presented in the section on attraction models.

Forward and Backward Linkages in the Theory of Economic Development

From the balance equation (in value units)⁸

$$z_i = m_i + q_i = \sum_j a_{ij} q_j + f_i^* \quad i = 1, \dots, n \quad (8)$$

and a linear relation, over a certain range, between m_i and q_i

$$m_i = \mu_i q_i \quad i = 1, \dots, n \quad (9)$$

there results an input-output system for a national economy:

$$q_i - \sum_j a_{ij} (1 + \mu_i)^{-1} q_j = (1 + \mu_i)^{-1} f_i^* \quad i = 1, \dots, n \quad (10)$$

If we write (10) in matrix notation as

$$\underline{q} - \hat{m}^{-1} A \underline{q} = \hat{m}^{-1} \underline{f}^* \quad (11)$$

A being a square and \hat{m} a diagonal matrix, it follows from (11) that

$$\underline{q} = (I - \hat{m}^{-1} A)^{-1} \hat{m}^{-1} \underline{f}^* \triangleq W \underline{f}^* \quad (12)$$

Chenery and Watanabe [4] have introduced the following linkage indexes:

$$\text{index of backward linkage: } \sum_i a_{ij} q_j / q_j = \sum_i a_{ij}$$

$$\text{index of forward linkage: } \sum_j a_{ij} q_j / z_i$$

Although both authors have been pioneers in the field of interindustrial interdependences, Laumas and Soper [10:285] point out several weaknesses in their approach.

From Rasmussen [15:13-14] the following indexes can be quoted:

$$\text{index of backward linkage: } U_{.j} = \frac{1}{n} \sum_i w_{ij} / \frac{1}{n_2} \sum_i \sum_j w_{ij}$$

⁸The symbols to be used have the following meanings: z_i = supply of commodity i ; m_i = imports of commodity i ; q_i = production of commodity i ; f_i = final demand for commodity i . Starred variables are exogenous.

$$\text{index of forward linkage: } U_{.i} = \frac{1}{n} \sum_j w_{ij} / \frac{1}{n_2} \sum_i \sum_j w_{ij}$$

where w_{ij} is the characteristic element of matrix W .

To interpret the index of backward linkage, matrix W is post-multiplied by vector $\underline{\Delta f}^*$:

$$\underline{\Delta f}^* = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \rightarrow j^{\text{th}} \text{ element} \quad (13)$$

Now the following equation holds (see (12)):

$$\underline{\Delta q} = \underline{\Delta} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta q_n \end{bmatrix} = \begin{bmatrix} w_{1j} \\ w_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ w_{nj} \end{bmatrix} \quad (14)$$

So, an increase in the final demand from industry J by one unit causes the total production of industry 1 to rise by w_{1j} , that of industry 2 by w_{2j} , etc., average production increase by industry

being equal to $\bar{w}_{.j} \triangleq 1/n \sum_i w_{ij}$. This average increase can be calculated for all values of j , i.e., for $j = 1, 2, \dots, n$. From the n averages, an overall average \bar{w} , can be determined, equal to $\bar{w} = 1/n \sum_j \bar{w}_{.j} = 1/n^2 \sum_i \sum_j w_{ij}$.

The Rasmussen index of backward linkage for industry j , then, is equal to 1 if $\bar{w}_{.j} = \bar{w}$; as a rule $\bar{w}_{.j} \neq \bar{w}$ and hence $U_{.j} \neq 1$.

For a good understanding of the structure of the index of forward linkage, matrix W is post-multiplied by vector Δf^* :

$$\Delta f^* = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (15)$$

Vector Δq reads as follows:

$$\Delta q^1 = \Delta \begin{bmatrix} \Delta q_1^1 \\ \cdot \\ \Delta q_i^1 \\ \cdot \\ \Delta q_n^1 \end{bmatrix} = \begin{bmatrix} w_{11} \\ \cdot \\ w_{i1} \\ \cdot \\ w_{n1} \end{bmatrix} \quad (16)$$

where superscript 1 indicates that the production increase concerned is due to an increase in the final demand from industry 1 by one unit.

Next, W is post-multiplied in succession by the vectors

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and the average of the production rises $\Delta q_1^1, \Delta q_i^2, \dots, \Delta q_i^n$, computed as $\bar{w}_i = 1/r \Delta w_{ij}$.

So, if the final demand for all industries rises by one unit, the average production increase of industry i is equal to \bar{w}_i . The overall average \bar{w} is then equal to the average of the series $\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n$, so $\bar{w} = 1/n \sum \bar{w}_i$.

The Rasmussen index of forward linkage for industry i is equal to 1 if $\bar{w}_i = \bar{w}$; as a rule again $\bar{w}_i \neq \bar{w}$ and hence $U_i \neq 1$.

Laumas and Soper [10:287] describe the meaning of $U_i \neq 1$ and $U_i \neq 1$ as follows: "If $U_i > 1$ it would mean that an industry draws heavily on the rest of the system, and vice versa if $U_i < 1$. Again if $U_i > 1$ it indicates that industry i would have to increase its output more than others for a unit increase in the final demand for all other industries, and vice versa if $U_i < 1$."

According to the numerical values of the backward or forward linkages, four groups of industries can be distinguished

Group I: strong forward and strong backward linkages;

Group II: weak forward and strong backward linkages;

Group III: strong forward and weak backward linkages;

Group IV: weak forward and weak backward linkages.

Table 5 gives a survey of the names found in literature for these groups.

Table 5

Indices	Chenery-Watanabe Indices [17]	Rasmussen Indices [10]
Group I	intermediate manufacture master industries industrie clef	leading sectors
Group II	final manufacture	backward-linked sectors
Group III	intermediate primary production	forward-linked sectors
Group IV	final primary production	unlinked sectors

These classifications are surely useful, but can hardly be integrated in models intended to compute the effects listed.

Forward Linkages in Regional Input-Output Analysis

We now present critically a first way of building forward linkages into an open input-output model for a region. The model from which Oosterhaven [12] has developed his model with built-in forward linkages reads as follows:

$$q_i^k = \sum_j a_{ij}^{kk} q_j^k + (f_i^{kk})^* + (x_i^k)^* \quad i = 1, \dots, n \quad (17)$$

where

- q_i^k = production of commodity i in region k (endogenous);
 f_i^{kk} = delivery of commodity i produced in region k for final use in that region (exogenous);
 x_i^k = exports of commodity i from region k (exogenous).

Equation (17) has been derived as follows: for region k , analogously to (8), the balance equation:

$$m_i^k + q_i^k \equiv \sum_j q_{ij}^{ok} + f_i^{oi} + x_i^k \quad i = 1, \dots, n \quad (18)$$

holds, where

- m_i^k = imports of commodity i into region k ;
 q_{ij}^{ok} = demand of industry j in region k for commodity i , wherever produced;
 f_i^{ok} = final demand for commodity i in region k , whatever the origin of the commodity.

The imports of commodity i into region k are equal to

$$m_i^k \equiv \sum_j q_{ij}^{.k} + f_i^{.k} \quad (19)$$

where

- $q_{ij}^{.k}$ = demand of industry j in region k for commodity i produced in other regions than k ;
 $f_i^{.k}$ = demand for commodity i produced in other regions than k for final use in region k .

Combining (18) and (19) results in

$$q_i^k = \sum_j (q_{ij}^{ok} - q_{ij}^{.k}) + f_i^{ok} - f_i^{.k} + x_i^k = \sum_j q_{ij}^{kk} + f_i^{kk} + x_i^k = \sum_j a_{ij}^{kk} q_j^k + (f_i^{kk})^* + (x_i^k)^* \quad (20)$$

where

- q_{ij}^{kk} = demand for products i produced in region k by industry j located in region k .

We do not take (17) as the starting point for our exercise, because in that model Full-Information technical coefficients (a_{ij}^{kk}) are employed while for reasons of statistical availability we strongly prefer using Limited-Information technical coefficients (a_{ij}^{ok}) in an input-output framework [14:272-273].

The open I/O model for a region into which we want to build forward linkages follows immediately from (18):

$$q_i^k + \tilde{m}_i^k \equiv \sum_j a_{ij}^{ok} q_j^k + (f_i^{ok})^* \quad i = 1, \dots, n \quad (21)$$

where

$$\tilde{m}_i^k \triangleq m_i^k - x_i^k \quad (22)$$

System (21) consists of n equations, but there are three n variables: n exogenous demands, n endogenous production levels, and n net import levels. Because we are not now interested in solving these equations for given final demands, we shall not make an assumption about the status of the variables m .

Keeping in mind the quotation from Oosterhaven in the introduction to this article, let us consider a situation in which the production of industry n cannot be imported. The *short delay* in response to an increase of:

- the final demand for the products or services of that sector;
- the intermediate demand due to an increase in the final demand in one or more other sectors;

while the nature and quality of the goods or services produced is such that they can be exported but cannot be imported. The production of industry n in region k , q_n^k , is therefore, at short term, exogenous.

With respect to the balance equation for industry n in region k

$$m_n^k + q_n^k \equiv \sum_j q_{nj}^{ok} + f_n^{ok} + x_n^k \quad (23)$$

one assumption is then that $m_n^k = 0$. From (19) and this last relation there follows that $\forall j$, $q_{nj}^{.k} = 0$ and $f_n^{.k} = 0$. Substitution of these relations into $q_{nj}^{ok} \equiv q_{nj}^{.k} + q_{nj}^{kk}$ and $f_i^{ok} \equiv f_i^{.k} + f_i^{kk}$ results in $q_{nj}^{ok} = q_{nj}^{kk}$ and $f_n^{ok} = f_n^{kk}$.

Equation (23) - in consideration of the exogenous character of q_n^k and f_n^{kk} - can now be rewritten as

$$(q_n^k)^* \equiv \sum_j q_{nj}^{kk} + (f_n^{kk}) + x_n^k \quad (24)$$

Because on the one hand $q_{nj}^{kk} = a_{nj}^{kk} q_j^k$ and $q_{nj}^{ok} = a_{nj}^{ok} q_j^k$, and on the other $q_{nj}^{ok} = q_{nj}^{kk}$, it is also true that $a_{nj}^{ok} = a_{nj}^{kk}$.

We are not going into the reasons why the production of industry n cannot be expanded at short delay, but will assume that in the situation described the difference between q_n^k and f_n^{kk} will be divided among a number of regular customers, located both within and outside region k ; in other words, the regular customers will be "offered" a portion of the production of industry n . In that case there exists a forward linkage: the supply of industry n , being the scarcest input factor, determines the production volume of the buyers.

Now let us assume that the sectors have been ordered in such a way that the commodities produced by industry n are delivered not to the first n_1 sectors, but to sectors $n_1 + 1, n_1 + 2, \dots, n-1$. It is then true that $q_{ni}^{kk} = b_{ni}^{kk} q_n^k$ and $q_{ni}^{ok} = a_{ni}^{ok} q_i^k$, b_{ni}^{kk} representing the sales or allocation coefficients. The forward linkages can now be expressed as follows:

$$q_i^k = \frac{b_{ni}^{kk}}{a_{ni}^{ok}} (q_n^k)^* \quad i = n_1 + 1, \dots, n-1 \quad (25)$$

Besides (24) and (25), our model with built-in forward linkages will contain the balance equation

$$q_i^k + \widetilde{m}_i^k \equiv \sum_{j=1}^{n-1} a_{ij}^{ok} q_j^k + a_{in}^{ok} (q_n^k)^* + (f_i^{ok})^* \quad i = 1, \dots, n-1 \quad (26)$$

Expression (24), written as

$$(q_n^k)^* \equiv \sum_{j=n_1+1}^{n-1} a_{nj}^{ok} q_j^k + (f_n^{ok})^* + x_n^k \quad (24')$$

combines with (25) and (26) into a system consisting of $2n - n_1 - 1$ equations. In this system the variables q_i^k, \widetilde{m}_i^k ($i = 1, \dots, n-1$), and x_n^k are endogenous, so the number of endogenous variables is n_1 more than the number of equations; to complete our model we add to (24'), (25) and (26) n_1 equations representing the assumed relation between \widetilde{m}_i^k and q_i^k for the first n_1 sectors:

$$\widetilde{m}_i^k = \mu_i^k q_i^k \quad i = 1, \dots, n_1 \quad (27)$$

It appears from our model that the net imports of the sectors to which a portion of industry n 's production is "offered" are endogenously determined; for the remaining sectors an import equation has to be specified. The old well-known problem of regional I/O analysis once more makes its appearance; viz., that of the stability of the import coefficients. If these coefficients are unstable through time, the forecasts of changes in the endogenous variables under given changes in the exogenous variables will be inaccurate.

From the reduced form of the model consisting of the structural equations (24'), (25), (26), and (27), the final effects of a given market structure of the output of industry n can be inferred.

If we write (24') and (25) through (27) in vector-matrix notation as follows:

$$(q_n^k)^* \equiv \underline{a}_3' \underline{q}_2 + (f_n^{ok})^* + x_n^k \quad (24'')$$

$$\underline{q}_2 = \underline{g} (q_n^k)^* \quad (25')$$

$$\underline{q}_1 + \underline{\widetilde{m}}_1 = A_{11} \underline{q}_1 + A_{12} \underline{q}_2 + \underline{a}_1 (q_n^k)^* + \underline{f}_1^* \quad (26a')$$

$$\underline{q}_2 + \underline{\widetilde{m}}_2 = A_{21} \underline{q}_1 + A_{22} \underline{q}_2 + \underline{a}_2 (q_n^k)^* + \underline{f}_2^* \quad (26b')$$

$$\underline{\widetilde{m}}_1 = \underline{\hat{m}} \underline{q}_1 \quad (27')$$

where

$$A \triangleq \begin{bmatrix} A_{11} & A_{12} & \underline{a}_1 \\ A_{21} & A_{22} & \underline{a}_2 \\ \underline{g}' & \underline{a}_3' & 0 \end{bmatrix}, \quad \underline{q}_1 \triangleq \begin{bmatrix} q_1^k \\ \vdots \\ q_{n_1}^k \end{bmatrix}, \quad \underline{q}_2 \triangleq \begin{bmatrix} q_{n_1+1}^k \\ \vdots \\ q_{n-1}^k \end{bmatrix}$$

$$\underline{g} \triangleq \begin{bmatrix} b_{n n_1+1}^{kk}/a_{n n_1+1}^{ok} \\ \vdots \\ b_{n n-1}^{kk}/a_{n n-1}^{ok} \end{bmatrix}, \quad \underline{\tilde{m}}_1 \triangleq \begin{bmatrix} \tilde{m}_1^k \\ \vdots \\ \tilde{m}_{n_1}^k \end{bmatrix}, \quad \underline{\tilde{m}}_2 \triangleq \begin{bmatrix} \tilde{m}_{n_1+1}^k \\ \vdots \\ \tilde{m}_{n-1}^k \end{bmatrix}$$

then (24'') and (25') through (27) can in turn be written as

$$P\underline{y} = Q\underline{z}^* \quad (28)$$

where

$$P \triangleq \begin{bmatrix} I - A_{11} & -A_{12} & I & O & O \\ -A_{21} & I - A_{22} & O & I & O \\ O & I & O & O & O \\ -\hat{m} & O & I & O & O \\ \underline{o}' & -a_3' & \underline{o}' & \underline{o}' & -1 \end{bmatrix}, \quad Q \triangleq \begin{bmatrix} I & O & O & \underline{a}_1 \\ O & I & O & \underline{a}_2 \\ O & O & O & \underline{g} \\ O & O & O & \underline{o} \\ O & O & 1 & -1 \end{bmatrix}$$

$$\underline{y} \triangleq \begin{bmatrix} q_1 \\ q_2 \\ \tilde{m}_1 \\ \tilde{m}_2 \\ x_n^k \end{bmatrix} \quad \text{and} \quad \underline{z}^* \triangleq \begin{bmatrix} \underline{f}_1^* \\ \underline{f}_2^* \\ (f_n^{ok})^* \\ (q_n^k)^* \end{bmatrix}$$

From (28) follows the reduced form of our model:

$$\underline{y} = P^{-1}Q\underline{z}^* \quad (29)$$

A Generalization

In an open input-output model for region k, apart from the balance equation (18), the definitional equation

$$v_j^k \triangleq q_j^k - \sum q_{ij}^{ok} \quad j=1, \dots, n \quad (30)$$

also holds, where

v_j^k = the value added of industry j in region k.

Symbol q_{ij}^{ok} stands for demand of industry j in region k for commodity i. Suppose that demand satisfied; then q_{ij}^{ok} also stands for the delivery of the products of industry i to industry j in region k. Now let us consider a situation in which the sales of the products of industry i to customers in region k as a fraction of the production of industry i in region k are fairly stable; on that assumption we specify, next to the demand relation

$$q_{ij}^{ok} = a_{ij}^{ok} q_j^k \quad (31)$$

the supply relation

$$q_{ij}^{ok} = b_{ij}^{ok} q_i^k \quad (32)$$

coefficient b_{ij}^{ok} being a Limited-Information allocation coefficient. From (31) and (32) there results that

$$q_i^k = \frac{b_{ni}}{a_{ni}} q_n^k \quad (33)$$

In contrast to (25), in (33) q_n^k is no longer exogenous; substitution of (32) into (30) gives

$$v_j^k = q_j^k - \sum_i b_{ij}^{ok} q_i^k \quad j=1, \dots, n \quad (34)$$

A structural model composed of (21) and (34) contains 2 n equations and 3 n endogenous variables; to complete the model, we specify a relation representing the share of v_j^k in $\sum_j v_j^k$:

$$v_j^k = \delta_j^d \sum_j v_j^k \equiv \delta_j^k (\sum_i (f_i^{ok})^* - \sum_i \tilde{m}_i^k) \quad j=1, \dots, n \quad (35)$$

where δ_k^d are the sectoral shares in total regional product, the latter being identically equal to total final demand minus net imports.

In vector-matrix notation the model reads:

$$\underline{q} + \underline{\tilde{m}} = A\underline{q} + \underline{f}^* \quad (21')$$

$$\underline{v} = \underline{q} - B'\underline{q} \quad (34')$$

$$\underline{v} = \underline{\delta} \underline{v}' (\underline{f}^* - \underline{\tilde{m}}) \quad (35')$$

After the substitution of (35') into (34') we write finally:

$$\begin{bmatrix} \underline{q} \\ \underline{\tilde{m}} \end{bmatrix} = \begin{bmatrix} I - B' & \underline{\delta} & \underline{v}' \\ I - A & & I \end{bmatrix}^{-1} \begin{bmatrix} \underline{\delta} & \underline{v}' \\ & I \end{bmatrix} \underline{f}^* \quad (36)$$

System (36) can be looked upon as a generalization of (29); however, it strongly rests on the assumption of stable allocation coefficients, as was stated earlier. That stability condition is a strong one; to reach a result like (36) without it, a combination of input-output and statistical regression can be used, as will be shown hereafter.

Attraction Models

Harry Richardson [16:185] observes the following:

- A serious weakness of input-output models as a methodological tool for spatial analysis is that they are non-spatial; one way of dealing with the problem is converting them to locational attraction models;
- The key hypotheses of attraction theory are that communication costs are a function of distance, that they are becoming more important in explaining locational distribution, and that high communication costs result in the agglomeration of complementary activities (locational attraction);
- The technical content of the model is to extend input-output analysis, with its focus on interindustry linkages on the demand side, to deal with attraction between consumers and industries and, more importantly, to introduce locational attraction on the *supply side* by measures of the relative importance of communication costs between sectors.

Attraction models are sometimes described as input-output models with built-in forward linkages. In this section we shall point out some agreements and differences between the input-output model with built-in forward linkages presented in the previous section and the attraction models.

Forward Linkages in Attraction Theory

The first step towards developing an attraction model is the definition of "communication costs" for a given industry; generally speaking, these costs consist of the efforts involved in selling the industry's production on output markets and acquiring the inputs it needs on input markets. If these markets are partly situated within a given region k , the assumption is made that the communication costs within region k , both per unit of product to be sold and per unit of input to be acquired, tend toward zero; on that assumption the communication costs for industry i in region k are defined as the sum of the communication costs involved in exporting a portion of the production and the communication costs resulting from the importation of the required inputs.

Both Klaassen [8:185], the founder of attraction theory, and Van Wickeren [18] define the exports of industry i in region k by means of (18). To that end (18) is written as

$$q_i^k \triangleq d_i^{ok} + \tilde{x}_i^k \quad i = 1, \dots, n \quad (37)$$

where

$$d_i^{ok} \triangleq \sum_j q_{ij}^{ok} + (f_i^{ok})^* \quad (38)$$

and

$$\tilde{x}_i^k \triangleq x_i^k - m_i^k \quad (39)$$

Symbol d_i^{ok} represents the demand for commodity i in region k , and \tilde{x}_i^k stands for net exports.

Other authors, among them Deacon [5], point out that by including net exports in the definition equation for the communication costs for industry i in region k , these costs are underestimated. These authors define the exports with the help of (20), written to that end as

$$q_i^k \triangleq d_i^{kk} + x_i^k \quad i = 1, \dots, n \quad (40)$$

where

$$d_i^{kk} \triangleq \sum_j q_{ij}^{kk} + (f_i^{kk})^* \quad (41)$$

The imports of commodity j by industry i in region k , q_{ji}^k , are equal to

$$q_{ji}^k \equiv q_{ji}^{ok} - q_{ji}^{kk} = a_{ji}^{ok} q_i^k - q_{ji}^{kk} \quad (42)$$

A question discussed above was for how long the specification of import equations could be put off, given the need to counter stability problems with respect to the import coefficients. In developing attraction models there is no such approach. One of the characteristics of attraction theory is that the quotient of the supply due to the demand of industry i in region k for commodities of industry j produced in that region, q_{ji}^{kk} , and the production of industry j in region k , q_j^k , is stable to some extent; in other words, the intermediate sales of industry j in region k have a structure such that

$$q_{ji}^{kk} = b_{ji}^{kk} q_j^k \quad (43)$$

where b_{ji}^{kk} is again the sales coefficient or "allocation coefficient."

According to that theory the left-hand part of equation (43) represents the supply effect of the production of industry j in region k ; on that basis (43) is described as the relation reproducing the forward linkages in attraction theory. Comparing the nature of these linkages with that of the forward linkages discussed above, however, we find an important difference: in equation (25) q_n^k is exogenous, in (43) q_j^k is endogenous. The forward linkage (25) determines the production volume of the industries using the products of industry n in region k as inputs; the forward linkage (43) does not in itself determine the production of industry i in region k , because that industry imports part of the total input q_{ji}^{ok} .

The Model

From (37) through (41) it already appears that there is no such thing as the attraction model. We shall not go into the differences and similarities of the various types of attraction models, nor elaborate on the underestimation of communication costs that is the result of using (39) in the equation for that quantity, but will conclude this section in the same way as the previous one, by trying to find out whether a number of structural equations constitute a complete attraction model.

The starting point is always the definition equation of the communication costs for industry i in region k :

$$c_i^k \triangleq \tau_i \bar{x}_i^k + \sum_j \tau_{ji} q_{ji}^k \quad i = 1, \dots, n \quad (44)$$

where

c_i^k = the communication costs for industry i in region k .

Combining (37), (38), (42), (43), and (44) results in:

$$c_i^k \triangleq (\tau_i + \sum_j \tau_{ji} a_{ji}^{ok}) q_i^k - \sum_j \tau_{ji} a_{ji}^{ok} q_j^k - \sum_j \tau_{ji} b_{ji}^{kk} q_j^k - \tau_i (f_i^{ok})^* \quad (45)$$

Klaassen [8:112] poses that the communication costs for industry i in region k can be looked upon as the value of the services rendered by the transportation and communication industries in region k to industry i in the same region; evidently Klaassen assumes that industry i in region k does not import any transportation and communication services. Thus

$$c_i^k \triangleq (\tau_i + \sum_{j \neq t} \tau_{ji} a_{ji}^{ok}) q_i^k - \sum_j \tau_{ji} a_{ji}^{ok} q_j^k - \sum_{j \neq t} \tau_{ji} b_{ji}^{kk} q_j^k - \tau_i (f_i^{ok})^* \quad (45')$$

The communication costs for industry i in region k are written as the product of the allocation coefficient b_{ti}^{kk} and the production of the transportation and communication industry in region k , q_t^k ; in symbols:

$$c_i^k = b_{ti}^{kk} q_t^k \quad i = 1, \dots, n \quad (46)$$

Equations (45') and (46) are structural equations of the attraction model developed by Klaassen; the variables c_i^k and q_i^k are endogenous; f_i^{ok} is exogenous. The number of endogenous variables is, like the number of equations, equal to $2n$; therefore, the model is complete. It can be proved, however, that the model is not identified.

Analogously to the procedure used above, the ultimate effect of the existing forward and backward linkages on the endogenous variables could be determined with the help of the reduced form of the structural model; that approach has not been followed by any author yet. The reason is obvious: the parameters of the reduced-form equation for c_i^k cannot be estimated, data on the communication costs not being available in practice. All authors, both those who base themselves on (37) and (38) and those who start from (40) and (41), take another road: after the formulation of a definition equation for the communication costs, a so-called attraction equation is formulated; the next step is to transform it into a relation between an industry's production and final demand in all sectors. The parameters in that relation are elements of the so-called attraction matrix; for details, the reader is referred to the literature [6].

Conclusions

Supply effects are essential in spatial economic analysis; since Weberian times they have been included in partial location analysis [14:Ch.3] and our operational models should endeavour to include them.

How they might be specified has been shown in the previous pages; spatial econometrics [13] should allow of estimating and testing them. Results have already been obtained, but work is going on in the field to derive more efficient estimating and testing procedures; this is a necessary condition for making our models trustworthy tools in preparing consistent regional policies.

One major example is the development of the so-called "FLEUR" model, a multisectoral (some 55 sectors) multiregional (some 70 regions) model built for the Common Market authorities. The results obtained integrate some of the previously exposed developments, and the extremely high correlations obtained (most were over .90!) demonstrate the efficiency of combining good regional statistics and an adequate model specification [1]

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