COMMENT/COMMENTAIRE

SIGNIFICANCE OF INTERREGIONAL FEEDBACKS FOR CANADIAN AND REGIONAL ENERGY POLICY DECISIONS: A COMMENT

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Introduction

In their research note, Douglas and MacMillan [3] provide interesting empirical evidence on the importance of an interregional or multiregional input-output model when the problem is to quantify impacts in several regions caused by exogenous increases in final demand *in several regions.* Their results illustrate that interregional *spillovers* can be large; for example, there is an increase in Ontario and Quebec incomes because of final demand increases in Alberta. What their study does not do is provide quantitative evidence on the importance of interregional *feedbacks* in the 11-region Canadian economy, at least in terms of the way that these effects have been defined in the literature.[1]

Verbally, interregional feedback effects are the differences in output (or income, or employment) in a particular region that are found in an interregional or multiregional input-output model, as opposed to an input-output model for the region in isolation, when the exogenous change occurs in one or more final demand elements *for that region only.* For example, if there were an increase in demand for the outputs of several Alberta industries, and if one's interest is in quantifying the impact of this increase on the Alberta economy, then the difference

[1]See Isard and Kuenne [7:300] for what appears to be the first definition in the literature. This was later explored in Miller [8;9;10], Greytak [5;6] and Gillen and Guccione [4]. See also Oosterhaven [15: Section 4.2] for a discussion of the distinction.

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between the impact in a single-region (Alberta) input-output model and a two- (or more) region interregional (or multiregional) model is the interregional feedback. There is a good deal of empirical evidence that these interregional feedback effects may be relatively small - considerably less than five percent - in a wide variety of real-world situations.[2] This suggests that for these single-region impact questions a more complete input-output model with endogenous interregional linkages (interregional trade coefficients matrices) may not be worth the trouble and expense.

Measuring Interregional Feedbacks

Douglas and MacMillan suggest that the interregional feedback effects (in terms of total income) in Quebec, Ontario, and Alberta respectively, are 88, 49, and 2 percent and conclude: "As can be seen, the interregional feedback effects . . . are very significant in Ontario and Quebec . . ." [3:254]. Although it requires some matrix algebraic statements of input-output relationships, it is not difficult to see why the first two numbers are so large relative to those in other empirical studies of interregional feedbacks.

Briefly, for a two-region economy (regions L and M, where L is the region of interest and M may represent the rest of the nation), the basic model is:

$$(I - A^{LL})\, X_T^L - A^{LM} X_T^M = Y^L \qquad (1)$$

$$-A^{ML} X_T^L + (I - A^{MM})\, X_T^M = Y^M$$

where the subscript "T" denotes outputs in each region when a two-region interregional model is used.[3] Reading all X's and Y's as "changes in" and when $Y^L \neq 0$ but $Y^M = 0$, we find through substitution in (1) that:

$$[(I - A^{LL}) - A^{LM}(I - A^{MM})^{-1} A^{ML}]\, X_T^L = Y^L \qquad (2)$$

In a single-region model for region L alone:

$$(I - A^{LL})\, X_S^L = Y^L \qquad (3)$$

where the subscript "S" denotes the results in a single-region model.

[2]For a brief summary of interregional feedback effect results from seven such empirical studies, see Miller and Blair [13:Table 4-6:127]. In six of these seven studies, the average measure of interregional feedbacks was less than three percent (in models in which households were exogenous).

[3]The algebra underlying the interregional feedback effect discussions has been presented in several places [8;9;10;4;13]. The intention in this paper is to present only as much as seems necessary to follow the argument and later derivations.

For some exogenous final demand change, Y^L, the left-hand sides of (2) and (3) can be equated, and thus, after rearrangement,

$$X_T^L - X_S^L = [(I - A^{LL})^{-1} A^{LM} (I - A^{MM})^{-1} A^{ML}]\, X_T^L \qquad (4)$$

Letting i be the summation vector (in the present case, a row vector of all ones), the overall percentage error (OPE) from ignoring the interregional feedbacks in the two-region model has been defined as:

$$\text{OPE} = \{i[X_T^L - X_S^L] \,/\, i\, X_T^L\} \times 100$$

Rather than *error*, this can also be thought of as the *extent* of interregional feedbacks; that is, the importance of these effects in a two-region input-output model. Gillen and Guccione [4] derived an upper bound for this error using relationships on the norm of a matrix (its largest column sum). Using $||M||$ for the norm of the matrix M, they show that:

$$||X_T^L - X_S^L|| \leq \frac{||A^{LM}||\; ||A^{ML}||}{(1 - ||A^{LL}||)\,(1 - ||A^{MM}||)}\, ||X_T^L|| \qquad (5)$$

For a nonnegative column vector, V, $iV = ||V||$, so:

$$\text{OPE} = [\,||X_T^L - X_S^L|| \,/\, ||X_T^L||\,] \times 100$$

Thus, from (5), the *maximum* value that the overall percentage error can have (MPE) is given in (6):

$$\text{MPE} = \left[\frac{||A^{LM}||\; ||A^{ML}||}{(1 - ||A^{LL}||)\,(1 - ||A^{MM}||)}\right] \times 100 \qquad (6)$$

It is to be emphasized that the interregional feedback error measures in OPE and MPE depend upon the assumption that $Y^M = 0$.

If there is a final demand change in both regions, so that both $Y^L \neq 0$ *and* $Y^M \neq 0$, then the relationships in (4) - (6) will all be altered. They become:

$$X_T^L - X_S^L = (I - A^{LL})^{-1} A^{LM} (I - A^{MM})^{-1} A^{ML} X_T^L + (I - A^{LL})^{-1} A^{LM} (I - A^{MM})^{-1} Y^M \qquad (7)$$

$$||X_T^L - X_S^L|| \leq \frac{||A^{LM}||\; ||A^{ML}||}{(1 - ||A^{LL}||)\,(1 - ||A^{MM}||)}\, ||X_T^L|| + \frac{||A^{LM}||}{(1 - ||A^{LL}||)\,(1 - ||A^{MM}||)}\, ||X^M|| \qquad (8)$$

$$MPE = \{ \frac{\|A^{LM}\|}{(1 - \|A^{LL}\|)(1 - \|A^{MM}\|)} [\|A^{ML}\| + \frac{\|Y^M\|}{\|X_T^L\|}] \} \times 100 \qquad (9)$$

It is the second term in brackets in (9) which is new, precisely because the Y^M vector is no longer null, and for any $\|Y^M\| > 0$ the upper bound in (9) will be larger than that in (6). The measure in (7), however, is simply not a measure of interregional feedback effects for region L, since it assumes that final demand has changed in both regions.

Some Scraps of Empirical Evidence

The Canadian linked-provinces model, like the U.S. linked-states model, is of the multiregional sort.[4] In Miller [10] it is suggested that the multiregional input-output (MRIO) version is particularly suited to analysis using the Gillen and Guccione upper bounds, as in (6). Also in that paper, in addition to results for two-region models, bounds are derived and numerical evidence is presented for three- and four-region input-output models, and it appears that there is generally not an increase in the MPE measure for comparable cases as the number of regions increases. Based on that evidence, plus results on spatial aggregation [14;1] suggesting that for region-L impact questions a two-region model (L = region of interest, M = rest of the nation) is about as good as one with more than two regions, consider comparisons of (6) and (9) for what may be "typical" Canadian cases.

To move from a pure interregional model to the U.S. MRIO model, a trade matrix like A^{LM} becomes $\hat{C}^{LM}A^M$, where the i^{th} element in the diagonal matrix $\hat{C}^{LM}$ represents the proportion of good i used as an input by producers in region M that is supplied by region L; the distinguishing feature of the MRIO model is that $\hat{C}^{LM}$ is diagonal; that is, that the proportion for input i supplied by L is constant across using

[4] The fundamental relationship in the MRIO model is given by $X = (I - CA)^{-1}CY$, where C is composed of the (diagonal) $\hat{C}$ matrices, A is block diagonal and contains each of the regional technical coefficients matrices, and X and Y contain gross outputs of and final demands for each sector in each region. The Canadian multi-provincial input-output model is in commodity-by-industry form, using industry-based technology; this complicates the notation slightly. Industry output of all sectors in each region (usually denoted X, or g) is given by $X = (I - DRB)^{-1}DRE$, where B is a matrix of commodity inputs per unit of industry output, R corresponds to C in the U.S. MRIO model, D is a matrix giving industry shares of commodity production, and E is a vector of final demands for each commodity in each region. In commodity-by-industry models, DB is one of the (several possible) matrices that plays a role similar to A in ordinary input-output models, DE is a conversion of commodity final demand to industry final demand, and DRB and DRE are the regionalized versions. Commodity output (usually denoted Q, or q) is given by $Q = (I - RBD)^{-1}RE$. For several alternative expressions of commodity-by-industry relationships and a discussion of industry-based vs. commodity-based technology, see Miller [11] or Miller and Blair [13:159-74].

industries in region M. Thus A^M is a regional technical coefficients matrix for region M, not a matrix of intraregional inputs to region M production. Similarly, A^{LL} in the interregional model becomes $\hat{C}^{LL}A^L$, and so forth. If α is a (scalar) measure of self-sufficiency for region L, then, roughly, $\hat{C}^{LL}A^L$ can be estimated as αA^L. In a two-region model, any domestic input not supplied internally must come from the other region. Thus if $A^{LL} = \alpha A^L$, $A^{ML} = (1 - \alpha)A^L$. Let β be a self-sufficiency measure for region M. Then, $A^{MM} = \beta A^M$ and $A^{LM} = (1 - \beta)A^M$. Since $\|\alpha A^L\| = \alpha\|A^L\|$, and similarly for the norms of the other A matrices in (6) and (9), these expressions become:

$$MPE = [\frac{(1 - \beta)\ \|A^M\|\ (1 - \alpha)\|\ A^L\|}{(1 - \alpha\ \|A^L\|)\ (1 - \beta\|A^M\|)}] \times 100 \qquad (6')$$

and

$$MPE = \{ \frac{(1 - \beta)\ \|A^M\|}{(1 - \alpha\ \|A^L\|)\ (1 - \beta\|A^M\|)} [(1 - \alpha)\|A^L\| + \frac{\|Y^M\|}{\|X_T^L\|}] \} \times 100 \qquad (9')$$

From the information in Douglas and MacMillan, it is possible to compare the values of the interregional feedback error measures in (6′) and the much broader measure in (9′), for three two-region Canadian models, with region L = Alberta, Ontario, or Quebec, and region M = rest of Canada in each case. Figures for $\|Y^M\|$ and $\|X_T^L\|$ are needed for (9′). The former are derived from Douglas and MacMillan, Table 1, column 1. Each final demand expenditure figure, $\|Y^M\|$, is obtained by subtracting the individual province (L) amount from the all-Canada total (3671, in mllions of 1974 dollars). Gross output figures, $\|X_T^L\|$, were provided by the authors.[5] They are 4478, 3827 and 1562, in millions of 1974 dollars, for Alberta, Ontario and Quebec, respectively. We consider the three two-region cases in turn.

Case 1. L = Alberta, M = Rest of Canada. According to Douglas and MacMillan, the measure of self-sufficiency for Alberta is α = 0.47. Empirical evidence in the U.S. MRIO model, with L = a particular state (Kansas or West Virginia) and M = rest of the U.S., suggests a level of self-sufficiency for region M of 0.99 or higher [12:Table 7]. Since Kansas or West Virginia may be less integral to the U.S. economy than Alberta, Ontario or Quebec is to the Canadian economy (but also since the Canadian model has households endogenous, which is not the case for the U.S. model just described), assume that β = 0.95. Evidence from two-, three- and four-region aggregations of the U.S. MRIO model, and a three-region aggregation of the Japanese IRIO model suggests that figures in the neighborhood of $\|A^L\|$ = 0.80 and

[5] Correspondence from Gordon W. Douglas, May 14, 1985.

$||A^M|| = 0.85$ are not wildly out of line.[6] For L = Alberta, we have $||Y^M|| = 1243$ and $||X_T^L|| = 4478$ (both in millions of 1974 Canadian dollars). In this case, the MPE figure in (6′), which is the upper bound for the traditional interregional feedback measure, is 15.01 percent; the figure given by (9′) is 24.84 percent. Since the Douglas and MacMillan feedback figure is 2 percent, not much is learned from the L = Alberta case.

Table 1

MAXIMUM PERCENTAGE ERRORS, WITH $||A^M|| = 0.85$

Assumed Values	Alberta ($\alpha = 0.47$) Eq. (6′)	Eq. (9′)	Ontario ($\alpha = 0.54$) Eq. (6′)	Eq. (9′)	Quebec ($\alpha = 0.47$) Eq. (6′)	Eq. (9′)
$\|\|A^L\|\| = 0.80$, $\beta = 0.95$	15.01	24.84	14.30	42.03	15.01	91.36
$\|\|A^L\|\| = 0.75$, $\beta = 0.95$	13.55	23.02	12.80	39.28	13.55	87.11
$\|\|A^L\|\| = 0.70$, $\beta = 0.95$	12.20	21.34	11.43	36.76	12.20	83.17
$\|\|A^L\|\| = 0.70$, $\beta = 0.90$	20.00	34.96	18.72	60.21	20.00	136.25

Case 2. L = Ontario, M = Rest of Canada. Again from Douglas and MacMillan, for Ontario, $\alpha = 0.54$, $||Y^M|| = 2730$ and $||X_T^L|| = 3827$. As in Case 1, let $||A^L|| = 0.80$, $||A^M|| = 0.85$ and $\beta = 0.95$; then the MPE from (6′) and (9′), respectively, are 14.30 and 42.03 percent. The bound in (9′) is now much larger than that in (6′), and Douglas and MacMillan's figure, 49 percent, is clearly much closer to the bound derived in (9′).

Case 3. L = Quebec, M = Rest of Canada. As for Alberta, $\alpha = 0.47$ for Quebec, $||Y^M|| = 3369$ and $||X_T^L|| = 1562$. With the same values for $||A^L||$, $||A^M||$ and β, the results in (6′) and (9′) are 15.01 and 91.36 percent, respectively. In this case, the Douglas and MacMillan figure for what they call the interregional feedback effect is 88 percent. Again, this is much closer to the bound provided by (9′) - 91 percent - than to the traditional interregional feedback measure, in (6′) - 15 percent.

Clearly, the MPE measures will be sensitive to the values assumed for $||A^L||$, $||A^M||$ and β. Table 1 illustrates this, for variations in $||A^L||$ and β. However, as indicated by several of the studies that have been referred to, the values that are assumed here appear to have a basis in empirical work. In any event, the important observation is that the larger the ratio $||Y^M|| / ||X_T^L||$ in (9′), the greater will the MPE derived from (9′), which captures spillovers, differ from its counterpart measure in (6′), which captures interregional feedback effects only.

[6]For two two-region U.S. cases, norm values range from 0.71 to 0.85 [12:249]. For the U.S. and Japanese three-region cases, the ranges are 0.55 - 0.60 and 0.59 - 0.86, respectively [13:80 and 66]. Values for the four-region U.S. case are in the range 0.64 - 0.71 (calculated from data in [2]). In all of these models, households are exogenous, and hence the matrix norms are undoubtedly less than they would be in a model, such as Douglas and MacMillan's, that is closed with respect to households.

Conclusion

To repeat, Douglas and MacMillan have convincingly shown the usefulness of a connected-regional input-output model in assessing the impacts in several regions caused by final demand changes in several regions of a national economy. In particular, they have shown that significant regional spillovers can occur. Their figures on the magnitudes of interregional feedback effects are exaggerated, however, simply because they measured the size of a different (and larger) animal. They are looking at giraffes, while the interregional feedback literature focuses on horses. Both creatures need to be measured, but they also need to be properly labelled.

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