DISTANCE DETERRENCE EFFECTS IN CONSTRAINED SPATIAL INTERACTION MODELS OF INTERPROVINCIAL MIGRATION*

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Introduction

Distance has long been viewed as a barrier to population migration, and there are numerous studies that identify the existence of a "distance decay effect" in migration and in other types of spatial interaction [5;8]. Traditionally the relationship between distance and migration has been examined with the use of the well-known gravity model, which is based on an analogy with Newtonian mechanics. Wilson, however, [32;33] was among the first to show that gravity-type interaction models can be derived, alternatively, from either statistical mechanics or information theoretic methods, utilizing the concept of entropy maximization. In general, the method of entropy maximization allows unbiased estimates to be made of the form of probability distributions, given limited prior information [18;21;30]. These estimates can then be employed in mathematical models of the system of interest that are to be tested against the observed data. In addition, the introduction of information in the form of constraints allows greater model flexibility and also overcomes the internal consistency problem inherent in the classical gravity model [13;20].

As is the case with classical gravity models, information theoretic entropy maximizing models emphasize the role of the distance variable

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in spatial interaction. In this regard, they have advantages over their classical predecessors, inasmuch as they allow explicitly for the incorporation of alternative functional forms for deterrence, or distance decay functions, in the modelling process [17]. This possibility presents a modelling framework that has application in the analysis of population migration.

This paper presents an empirical test of three versions of a totally constrained entropy maximizing model of population migration, which differ with respect to their distance deterrence functions. Three models—inverse power, negative exponential, and squared distance negative exponential—are tested in terms of their ability to replicate observed annual patterns of Canadian interprovincial migration during the period from 1961-62 to 1984-85. The intent of this paper is not only to test the empirical performance of the models but also to compare and contrast the results in terms of the ability of the models to provide insights into the role of distance in the migration process.

Within the context of urban and regional modelling, it is to be noted that as the relative magnitude of demographic change due to natural causes continues to stabilize [23], population migration, or the lack thereof, begins to play a greater role in reducing, or intensifying, urban and regional disparities and imbalances. The connection between regional adjustment and population migration has been made explicit in a recently published debate and discussion [21;22]. The perceived significance of the role of population migration in Canada is demonstrated by the high level of interest in the subject, and several modelling strategies have been proposed [6;10;12;15;22;34]. The modelling methodology proposed in the present study provides one additional avenue for the analysis of spatial patterns of population migration. It differs from those listed above in that the emphasis is placed, and deliberately so, on the role of the distance variable in modelling human migration. Discussions of the use of information theoretic models in the context of Canadian interprovincial migration are found in Ledent [11] and Pooler [19].

The Models and Deterrence Functions

The trip distribution models employed in this analysis are variants on the totally constrained entropy maximizing model. This well-known model is derived from entropy, subject to origin, destination and cost constraints on, respectively, the number of people leaving and entering each zone and the total distance travelled in the system:

$$T_{ij} = A_iO_iB_jD_j \exp(-\beta d_{ij}), \tag{1}$$

where T_{ij} is the predicted number of trips between zone i and j, O_i and D_j are the total number of trips leaving origin i and entering destination j, and A_i, B_j and β are parameters (also referred to as balancing factors) that ensure that the origin, destination and distance constraints, respectively, are met [18;20;32;33]. Equation (1) is a constrained interaction model which represents the statistically most probable arrangement of trips among n cells [33]. The A_i and B_j parameters are defined as:

$$A_i = [\Sigma_{j=1}^{n} B_jD_j\exp(-\beta d_{ij})]^{-1}, \tag{2}$$

and

$$B_j = [\Sigma_{i=1}^{n} A_iO_i\exp(-\beta d_{ij})]^{-1}. \tag{3}$$

The term A_i is the ratio between O_i and the sum of the *unscaled* predictions for all trips leaving zone i, and the term B_j is the ratio between D_j and the sum of the *unscaled* predictions for all trips entering zone j [28:149].

The totally constrained entropy maximizing model of spatial interaction, presented above, produces a negative exponential distance deterrence function when the distance constraint to be satisfied relates to linear distance. However, the researcher is not confined to this deterrence function or distance constraint. It is possible to specify other distance deterrence functions in the derivation by changing the nature of the constraint, see [2;9;29;30;32].

If the distance constraint is written instead with respect to the logarithm of distance, then the derivation produces an *inverse power* deterrence function (henceforth, called the *power* function) of the form, $d_{ij}^{-\beta}$ [17;29;30]. This model, which has the effect of decreasing the relative separation between origins and destinations, is written as follows:

$$T_{ij} = A_iO_iB_jD_jd_{ij}^{-\beta}, \tag{4}$$

where A_i is defined as:

$$A_i = [\Sigma_{j=1}^{n} B_jD_jd_{ij}^{-\beta}]^{-1}, \tag{5}$$

and B_j is defined as:

$$B_j = [\Sigma_{i=1}^{n} A_iO_id_{ij}^{-\beta}]^{-1}. \tag{6}$$

To justify this model Wilson [33:35] noted:

> Suppose people perceive travel costs not as we measure them, but as the logarithm of what we measure. Such an assumption would apply if the cost of travelling 50 miles was perceived to be less by the traveller who was committed to 200 miles anyway than to the traveller who was going 50 miles in total. Then d_{ij} is replaced by ln d_{ij}, and

$\exp(-\beta d_{ij})$ becomes $\exp(-\beta \ln d_{ij})$, which is $d_{ij}^{-\beta}$. Thus, if models fit better with inverse power functions than with negative exponential functions, this tells us something about the way travellers perceive costs.

An alternative approach to defining a deterrence function is to write the distance constraint with respect to the square of distance (and the variance of the trip frequency distribution, see [1;29]). This approach to derivation produces an exponential deterrence function of the form $e^{-\beta d_{ij}^2}$, which "is proportional to a truncated normal distribution" [1:211]. The resulting interaction model, which we term the "squared distance exponential", has the following form:

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta d_{ij}^2), \qquad (7)$$

where A_i is defined as,

$$A_i = [\Sigma_{j=1}^{n} B_j D_j \exp(-\beta d_{ij}^2)]^{-1}, \qquad (8)$$

and B_j is defined as,

$$B_j = [\Sigma_{i=1}^{n} A_i O_i \exp(-\beta d_{ij}^2)]^{-1}. \qquad (9)$$

Although the *overall* amount of spatial interaction *decreases* with distance in this model, it is important to point out that the *marginal* friction of distance *increases* with distance; therefore this function works in exactly the opposite way to the power function. Given what is known from existing studies of population migration, this may be considered to be a somewhat inappropriate function with which to model population migration, but we have chosen to consider it briefly for two reasons. First, if this is an inappropriate form for a migration model, it is of interest to see just how inappropriate it is. Second, this model will indicate the extent to which the marginal friction of distance is increasing or decreasing through time. For example, will the performance of the squared distance exponential model improve or decline over the twenty-four-year study period? Squared distance exponential functions have been discussed previously in the context of spatial interaction modelling in Taylor [27] and in Batty and March [2].

The majority of past research employing information theoretic, spatial interaction models has focused on the use of negative exponential deterrence functions. Although alternative forms of the distance variable have been employed regularly in population migration modelling (for example [12]), there have been few previous attempts to employ the deterrence functions outlined above in the context of information theoretic models of population migration (one notable exception is [26]). The remainder of this paper presents the results of tests of the ability of these models to replicate observed spatial patterns of interprovincial population migration, using the three deterrence functions presented above.

Operational Aspects of the Model

The interprovincial migration data that are employed in this study are those derived from family allowance and income tax records by Statistics Canada [25]. They are estimates of the gross number of children and adults migrating annually among the provinces and territories during the period from 1961-62 to 1984-85 (a year of data covers the period between June 1st of one year and May 31st of the following year). The estimates of migration are discussed in detail in Statistics Canada, Catalogue 91-210 [25:20-27].[1]

This study uses road distances between major provincial cities.[2] Road distance, like great circle distance, is not an ideal measure of separation, and its shortcomings are recognized, although it should represent various concepts such as isolation, information flow, ease of access, cultural effects, and so on.

Two goodness-of-fit statistics are used to assess model performance. One is the percentage of migrations misallocated:

$$\text{Percent Misallocated} = \frac{50}{T} \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} \left| T_{ij} - T_{ij}^{*} \right|, \qquad (10)$$

where T is the total number of migrants, T_{ij} is the number of migrants moving from province i to province j, and T_{ij}^{*} is the number predicted.[3] Any value of the measure represents the percentage of all migrants who would have to be reallocated to the correct cells in order for the predicted matrix to match the observed matrix.

A second goodness-of-fit statistic, to be employed below in the discussion of spatial patterns of residuals, is the phi statistic:

$$\phi = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} \frac{T_{ij}}{T} \left| \ln \frac{T_{ij}}{T_{ij}^{*}} \right|. \qquad (11)$$

[1] Prior to 1976, for administrative purposes, the migration data were organized as eleven spatial units; ten provinces and the Yukon and Northwest Territories (N.W.T.). After 1976 the two territories were disaggregated by Statistics Canada to give a twelve-zone spatial system. For ease of comparison in the present study, the two territories are amalgamated after 1976 and treated as one separate unit as they were by Statistics Canada before that year.

[2] The distance between the amalgamated units of the Yukon and N.W.T. and the other provinces is taken as the average road distance (in miles) between the major provincial cities and Whitehorse and Yellowknife.

[3] In order that each misplaced migrant is not included in the calculation as both an over- and underestimation, only *half* of the total number misallocated is counted.

Phi is similar to the information gain measure [2] in which larger values represent poorer model performance.

Analysis of Model Performance

Overall Performance

Figure 1 shows the overall variation in percentage of migrants misallocated, with horizontal lines representing mean performance through the entire study period. In terms of the general performance of the three totally constrained models over the twenty-four-year period, the power model is superior with a mean value of 7.0 percent, followed by the negative exponential at 8.8 percent, and the squared distance exponential at 17.8 percent. It can be seen that while the negative exponential and power models produce similar results over the study period, the squared distance exponential, with its increasing marginal friction of distance, does not compare well in terms of goodness-of-fit. Although the performance levels of the negative exponential and power models remain very close through time, it is important to note that, with the exception of a brief period between 1963 and 1966, the power model outperforms consistently the negative exponential. These relative performance levels are not unrelated to the magnitude of the mean distances travelled by migrants through the study period, as will be discussed in greater detail below.

Figure 2 shows, for each of the three models, the mean percentages of observed and predicted migrants assigned to five distance categories for the entire study period. The distance categories were chosen to represent the range of distances moved within the system from short (<500 miles) to very long (over 3000 miles). In terms of performance with respect to the distance categories, it can be seen that the power model is again the one that is most consistent with the overall pattern of assignment. Although the power model tends to produce slight overpredictions in shortest and longest distance categories, it performs fairly consistently in the three middle ranges, underpredicting the flows by a small margin in each case. The negative exponential model, on the other hand, shows just the opposite pattern. Migrations are underpredicted in the shortest and two longest distance categories and are overpredicted in the two middle ranges (500-1000 and 1000-2000). As expected, the tendency of the squared distance exponential model is to underpredict long distance moves (3000 plus and 2000-3000) and to overpredict shorter distance moves (1000-2000 and 500-1000). A notable exception to this expectation is to be found in the category representing moves of less than 500 miles, wherein the model produces a small underprediction. This unexpected

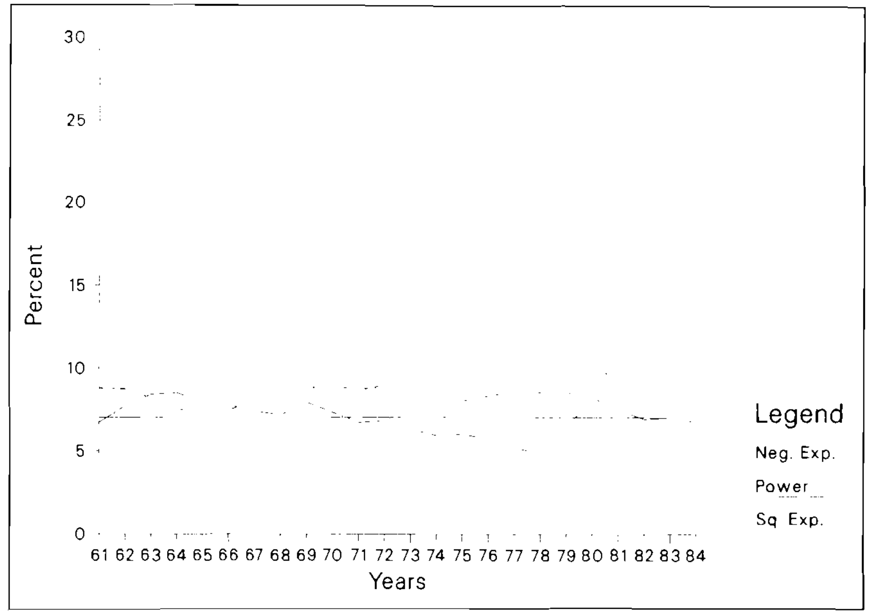

Figure 1

PERCENTAGE OF MIGRANTS MISALLOCATED BY THE THREE MODELS: 1961-1984

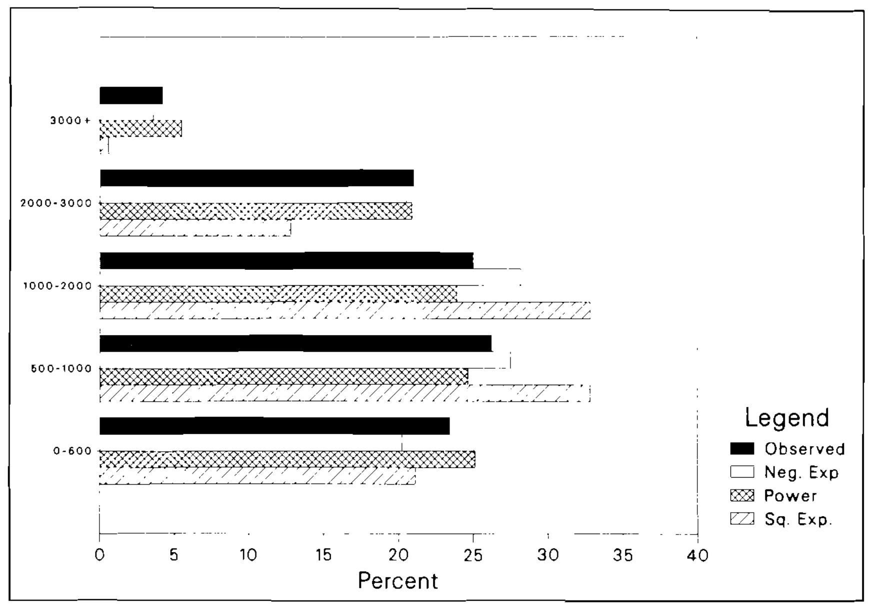

Figure 2

MEAN PERCENTAGE OF OBSERVED AND MODEL ASSIGNMENTS TO FIVE DISTANCE CATEGORIES: 1961-1984

result can be attributed, in this particular case, to the effects of the constraint on the mean distance travelled. Although it is the tendency of the model to predict more, shorter distance moves, nevertheless it must be calibrated to satisfy the constraint on the observed mean distance of migration. This has the effect of taking movers out of the shortest distance category and forcing them into the larger categories. Given this consideration of overall levels of performance, the remainder of this section discusses briefly the results for each of the three models and then goes on to discuss the spatial patterns of results and residuals.

The Squared Distance Exponential Model

From the onset of the study period, as expected the performance level of the squared distance exponential model generally declines, although it is most successful until 1969, before which time the percentage of migrants misallocated is below the average value. However, throughout the 1960s, the errors of the model become more significant as it attempts to assign fewer migrants to a distance category that is expanding (2000-3000 miles) and compensates by allocating more to a category that is declining (500-1000 miles).

The trend of deteriorating performance matches closely the observed increases in mean distance travelled, particularly after 1970 when an apparent threshold level of 1200 miles is exceeded (see Figure 3). Much of the reason for the falling performance level of the model is attributable, therefore, to the poor presentation of the marginal friction of distance in the deterrence function. The use of squared distances within an exponential function results clearly in an inadequate representation of the changing distribution of flows within the migration system.

The Power Model

The power model's performance also supports the idea of a threshold level of observed mean distance travelled (1200 miles) as being of relevance to this system of study. This threshold is the mean distance travelled in 1970, before which time the power model produces its poorest results. Between 1961 and 1969 an average of 3.2 percent of all migrants in the 500-1000 mile range are underpredicted. The increase in percentage misallocated between 1961 and 1963 (see Figure 1) is consistent with the observed increase in the percentage of the relative numbers of these migrants, after which their proportion declines. The percentage moved by the model in the 500-1000 mile distance category is stable between 1961 and 1969. The overall model performance, therefore, improves through this period of time as the

magnitude of predicted flows moves closer to the observed level. There are several reasons why model fit continues to improve through the 1970s, as the mean distance travelled increases (Figure 3). The response of the power model to these increases is to allocate more migrants to the 500-1000 mile category, taking them from the "less than 500 mile" category, within which the observed number of migrants is decreasing. The net effect is to decrease the difference in observed and predicted flows for these two categories that contributed to higher model error during the 1960s.

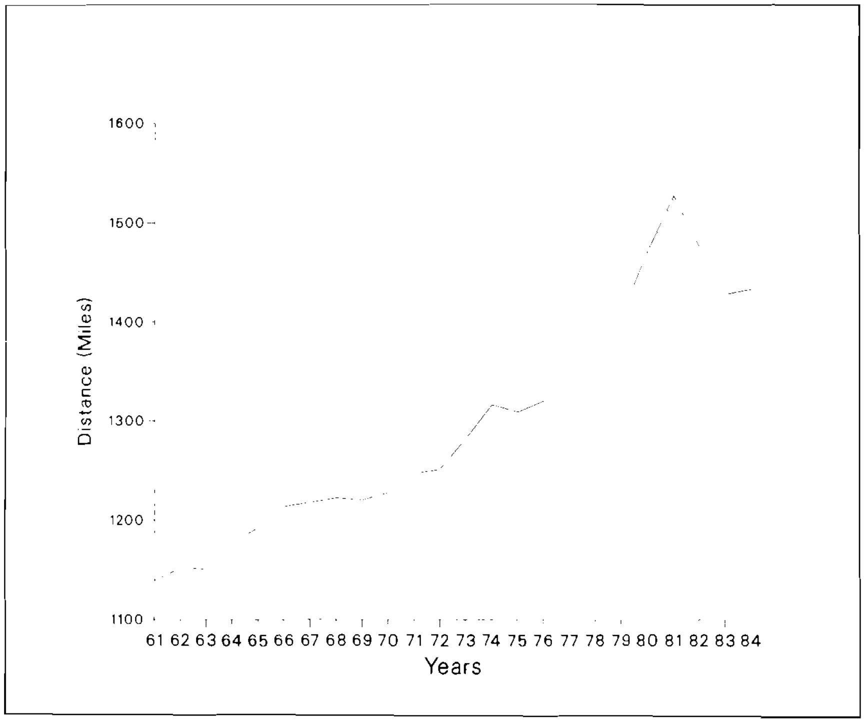

Figure 3

OBSERVED MEAN DISTANCE TRAVELLED (MILES): 1961-1984

Between 1978 and 1980 the model produces its best (4.8 percent) and third poorest (8.2 percent) overall performances. These results are in parallel with relatively large changes in observed mean distance travelled (from 1297 to 1354 miles) and changes in numbers of migrants (395,724 to 445,780) that occurred over the same period of time. The change in mean distance travelled is due to an increase in the greater proportion (29.3 percent compared to 24.5 percent) of people moving between 2000 and 3000 miles. This is reflected in the large number of migrants leaving Ontario, in particular, for Alberta, the percentage of which changed from 26.6 percent in 1978 to 36.8 percent in 1980. The model, however, fails to replicate this dominant flow over such a large distance (2147 miles) and predicts only an increase from 25.1 percent to 31.3 percent between the two years. The model responds instead by moving more migrants short distances from British Columbia and Saskatchewan to satisfy the destination constraint for Alberta. In this example the distance constraint fails to represent adequately the range of migrant opportunities in Alberta and its strong ties with Ontario. For example, many of the misplaced Alberta-bound Ontarians are moved to British Columbia, which is a destination for a large number of migrants, but most of these are from neighbouring provinces (nearly 50 percent of all Alberta emigrants were destined for British Columbia during 1980-81). Ironically, in terms of overall assignment to distance categories, the power model is most representative of the actual allocation of flows in 1980, although the "chain reaction" promoted by the large flows to Alberta from Ontario has had important consequences in regard to accurate migrant distribution among the cells of the matrix.

The Negative Exponential Model

The negative exponential model performs most effectively during the period 1961 to 1969, representing a range of mean distances travelled from 1116 to 1200 miles. During this period, the model represents best those migrants in the 500-1000 mile and 2000-3000 mile ranges, predicting 27.5 percent and 16.1 percent, respectively, compared to 26.8 percent and 16.3 percent of observed migrants. As the mean distance travelled exceeds the threshold distance of 1200 miles, model performance fluctuates around the mean percentage misallocated value until 1980 (see Figure 1). This deterioration in model performance is in response to the distance constraint, the effect of which forces the model (between 1970 and 1980) to underpredict short distance moves (18.9 percent compared to 22.3 percent observed in the "less than 500 mile" category) and to overpredict those moves in the 1000-2000 mile category (28.1 percent compared to 24.8 percent observed). Between 1974 and 1979, model performance is slightly better than average. The reason for this is the increase in the relative number of moves between 2000 and 3000 miles, which is most rapid after 1974 and related to the growing importance of Alberta at this time. The model's response to an observed increase in these moves from 19.1 percent in the period 1970-74 to 23.5 percent in 1976-79 is to represent more accurately the respective percentages of these moves, changing from

23 percent (an overprediction of 3.9 percent) in 1970-74 to 22.9 percent (an underprediction of .6 percent) in 1976-79.

Between 1979 and 1981 the negative exponential model assignment of migrants to distance categories is more accurate compared to the preceding three years, although the goodness-of-fit statistic records its highest percentage misallocated value in 1981 (10.2 percent). As with the power model, the poor performance during this period is related to the distribution of migrants among the cells of the predicted matrix, and not to the assignment to various distance groups. In fact the assignment to distance categories improves between 1979 and 1981 as a result of increases in the value of mean distance; this has the positive effect of increasing the proportion of moves over 2000 miles, and decreasing those below. Most of the goodness-of-fit problems during this period of time arise therefore from the matrix distribution of moves, particularly those below 1000 miles. For example, 25 percent of the *total* error is attributable to overprediction of flows from British Columbia to Alberta (5.9 percent) and underprediction of flows between Saskatchewan and Manitoba (8.5 percent) and from Quebec to Ontario (10.6 percent). There is some improvement in performance associated with the decrease in the value of the distance constraint in the last three years of the study for the negative exponential model as the observed proportion of moves below 2000 miles increases and those above decrease. It is this changing distribution of observed flows that is most suited to the behaviour of the negative exponential model.

Spatial Patterns and Residuals

In addition to the preceding consideration of the performance of the models over time, it is of interest to look at the *spatial* patterns of goodness-of-fit. Two approaches are taken with respect to these patterns: first, a matrix that measures mean performance levels over the twenty-four-year study period will be discussed; and second, the pattern of residuals for the year 1981 will be considered briefly as an example of an interesting, and perhaps atypical year in Canadian interprovincial migration. Inasmuch as the previous discussion demonstrated the superiority of the power model in forecasting migration flows, only the performance of that model will now be discussed.

Although a discussion of error averaged over time makes it difficult to suggest *empirical* sources of discrepancy between observed and predicted migration flows on a year-by-year basis, it serves, nevertheless to suggest where the most common sources of error occur in the predicted matrices. For each year of the study period, phi statistics (Equation (11)) were calculated as an overall measure of the difference between the observed and predicted matrices. Individual phi values, measuring error in the cells of the matrix, were then calculated as a percentage of phi. Table 1 presents the twenty-four-year totals of those individual phi statistics in each cell of the matrix.

It is important to note that these are the sums of the absolute values of the individual phi values and therefore do not distinguish between underprediction and overprediction; they represent an *overall* measure of the performance of the power model with respect to the flows between each pair of provinces. In order to facilitate the interpretation of these numbers, Table 2 presents a summary of the signs of the residuals for those pairs of provinces having the largest differences between observed and predicted flows (those with total errors of approximately 60 percent or greater, as indicated by the circled numbers in Table 1). Some interesting patterns emerge in the residuals. One of the largest sources of error is the predicted flows from Alberta to British Columbia. Table 2 shows that this dyad is underpredicted consistently until 1981. After that, an overprediction appears, presumably in response to large outflows associated with the end of the economic boom in Alberta. Prior to that time, it would appear that the model fails to capture the regional ties that exist between these two provinces.

The largest and most persistent dyadic error that results from the use of the power model is in the constant underprediction of flows in both directions between Saskatchewan and Manitoba. Our interpretation of this pattern is that outmigrants who might be expected to move between this pair of provinces move instead to the larger neighbours of Alberta and Ontario, where more economic opportunities are usually available.

As is usual in the analysis of patterns of interprovincial migration in Canada, the Ontario/Quebec dyad represents an exceptional pattern of flows, given the distances. Tables 1 and 2 indicate that the large error in the estimates of flows from Ontario to Quebec is due consistently to an overprediction of moves. Similarly the same pattern holds for the reverse flow, at least until the 1976 election year, after which the observed flow of migrants from Quebec to Ontario is underpredicted consistently. Clearly the "distance" between Quebec and Ontario is very different before and after 1976, for the purposes of population migration.

Other large sources of error in the model predictions are associated with the provincial pairs of Quebec/New Brunswick, Ontario/Nova Scotia and Ontario/Newfoundland. In all of these pairs, there is a consistent underprediction of flows in both directions for most of the study period. This suggests that there may be migratory and economic ties between these pairs of provinces that are not being captured by the distance variable in the power model. Chain migration—the tendency of migrants to follow previous migrants due to an

Table 1

TOTAL PERCENTAGE PHI ASSOCIATED WITH EACH DYAD: 1961-1984

	Origin	Destination										
		1	2	3	4	5	6	7	8	9	10	11
1	Yukon/NWT	0.00	13.21	19.15	3.04	2.36	12.84	2.45	1.06	0.54	1.09	1.96
2	British Columbia	15.74	0.00	44.31	20.93	18.47	26.63	23.86	9.80	6.39	14.62	14.33
3	Alberta	20.31	80.30	0.00	20.35	32.97	41.51	15.56	10.14	3.69	15.14	10.29
4	Saskatchewan	3.86	18.78	36.96	0.00	100.67	46.93	7.89	4.75	1.09	5.40	4.64
5	Manitoba	3.33	23.41	23.06	108.12	0.00	30.90	9.29	5.67	1.59	5.84	5.89
6	Ontario	13.67	34.89	52.67	36.57	21.15	0.00	83.28	39.09	16.74	76.76	59.07
7	Quebec	3.55	22.14	19.19	14.37	13.20	88.27	0.00	93.09	3.82	20.06	15.96
8	New Brunswick	1.37	17.04	9.69	4.19	5.11	38.06	99.27	0.00	4.30	50.56	2.42
9	PEI	0.39	3.33	4.60	0.58	1.52	11.74	2.30	4.54	0.00	7.44	1.63
10	Nova Scotia	1.47	8.81	12.39	5.24	4.65	84.91	6.91	53.65	9.92	0.00	13.88
11	Newfoundland	3.45	23.99	15.62	5.56	5.43	59.68	8.77	3.21	2.17	21.49	0.00

Table 2

SIGN OF THE MODEL RESIDUALS FOR MAJOR ERROR DYADS: 1961-1984

Largest Residuals	Year																							
	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Alberta to British Columbia	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+
Saskatchewan to Manitoba	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Manitoba to Saskatchewan	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Ontario to Quebec	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Quebec to Ontario	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
Ontario to Newfoundland	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Newfoundland to Ontario	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+
Quebec to New Brunswick	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
New Brunswick to Quebec	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Nova Scotia to Ontario	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+
Ontario to Nova Scotia	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+

information feedback process [7]—is suggested as a mechanism that may be at work here. The positive residuals for the last four years of the study period for the Ontario/Nova Scotia dyad seem to be a result of the fact that both of these provinces become more significant (that is, larger) destinations during this time.

One of the more interesting aspects of the spatial patterns of residuals resulting from the power model is that the largest errors occur consistently in both directions between matched pairs of provinces; the pattern of matrix errors, in other words, is largely symmetrical. This suggests that the model is misspecified most often, not with respect to individual, unidirectional migration flows, but rather with respect to dyadic pairs of flows and distances. Further attempts to improve model performance should take into account the dyadic nature of the model errors.

Overall, the most striking feature of the pattern of residuals in Table 2 is clearly the dominance of overpredictions between Quebec and Ontario, and in the underpredictions between almost all other pairs of provinces. Quebec and Ontario are very large origins and destinations, situated in close proximity to one another, and therefore the model predicts very large flows between them. It would appear from Table 2 that it is, in fact, these overpredictions that cause almost all of the other large residuals to be underpredictions. Clearly, models of interprovincial migration using distance as an explanatory variable need to recognize the unique location of Quebec in Canadian migration space [14].

As a second approach to the analysis of residuals, we decided to examine the unique case of 1981; the final year of the period during which Alberta was having extraordinarily large net population gains due to the economic boom occurring in the province.[4] Table 3 presents the matrix of phi values for the power model predictions in 1981 (errors greater than 3 percent are circled).

The two largest sources of error in this matrix are associated with the prediction of flows to Alberta; in the case of Ontario the flows are underpredicted, and in the case of British Columbia they are overpredicted. At this point in time, Alberta is a very large destination and consequently the model attempts to "fill the demand" for migrants in Alberta, by "supplying" them from British Columbia. The flow from Ontario is then underestimated, even though this is the source of a large portion of migrants entering Alberta. Similarly, another large error (greater than 3.00 percent) is in the underprediction of flows from British Columbia to Ontario. This suggests again that Alberta, as a large destination, is acting as an intervening opportunity in the

[4]In 1981-82 Alberta had a net gain of 36,562 migrants; by 1982-83 this had turned into a net loss of 11,650 [25].

Table 3

PERCENTAGE OF PHI ASSOCIATED WITH EACH DYAD FOR 1981

Origin		Destination 1	2	3	4	5	6	7	8	9	10	11
1	Yukon/NWT	0.00	-1.27	+0.91	-0.24	-0.04	+0.40	-0.14	+0.04	+0.01	-0.04	-0.11
2	British Columbia	-0.94	0.00	+7.36	-1.89	-1.60	-3.47	-1.13	+0.14	-0.02	-0.85	+0.51
3	Alberta	+0.92	-1.59	0.00	+0.79	+0.98	+1.51	-0.40	-0.84	-0.20	-1.43	-0.41
4	Saskatchewan	-0.07	-0.81	+1.86	0.00	-4.99	+1.34	+0.23	+0.10	-0.01	+0.14	+0.17
5	Manitoba	-0.14	-0.53	+2.29	-5.49	0.00	+1.05	+0.21	+0.20	+0.04	+0.36	-0.04
6	Ontario	+0.20	-0.41	-6.27	+2.06	+1.00	0.00	+0.85	+1.50	+0.21	+0.58	-0.66
7	Quebec	+0.17	+1.21	-1.71	+0.85	+1.15	-4.94	0.00	-1.65	+0.25	+1.44	+0.69
8	New Brunswick	+0.01	+0.82	-2.07	+0.23	+0.34	+1.56	-2.02	0.00	-0.17	+0.10	-0.01
9	PEI	-0.04	+0.26	-1.04	+0.06	+0.06	+0.62	+0.16	-0.37	0.00	-0.44	+0.03
10	Nova Scotia	-0.17	+0.27	-1.06	+0.33	+0.30	+0.89	+0.37	+0.05	-0.48	0.00	-1.00
11	Newfoundland	-0.48	+1.05	-2.63	+0.34	+0.33	-0.06	+0.54	+0.20	+0.08	-0.93	0.00

model, capturing migrants that would otherwise have been sent to Ontario. The other large residuals coincide to some extent with those discussed previously; the Quebec/Ontario and Saskatchewan/Manitoba pairs are again among the most difficult to predict. Overall, the 1981 model results indicate that a large economic boom in a single province can have the effect of distorting not only flows to and from the boom province but also flows to and from other provinces. In the boom period, Alberta has an impact on national patterns of migrant flow which is of a magnitude comparable to the usual impact of Quebec.

Discussion

The results described above demonstrate that the power function is most applicable in this particular study of population migration. However, it is important to note that the better performance of the power model is confined, in this system, to the *totally* constrained version and is a result of a combination of the *distance constraint* and the nature of the function itself. For example, *doubly* constrained versions of the negative exponential and the power model produce mean percentage misallocated values of 9.3 percent and 23.3 percent respectively for the twenty-four-year study period.[5] It is apparent that the ability of the *totally* constrained model to duplicate observed patterns of flow depends, in large part, on the nature of the distance constraint to be satisfied and on the related form of the distance deterrence function. One of the reasons for the superiority of the power function in this study is that, in relative terms, its distance constraint is particularly able to account for the changes in actual mean distance travelled. In other words, as the average distance travelled increases through time, the power model improves in performance because its distance constraint has the effect of forcing more migrants over greater distances.

The power function provides the best representation of distance in the migration model as it reduces the "frictional" effect on movement in a system where the observed patterns of movement show, over time, (1) an increase in mobility and, (2) an increasing tendency for migrants to move longer distances. Due to the large size of most provinces, a sizeable proportion of interprovincial moves can be described as "long" distance and, in this sense, Canadian migrants have always been "mobile". However, whatever the precise role of distance in the migration process, it represents a barrier of sorts, as the larger number of shorter distance moves among the provinces indicates. As

[5]The difference between doubly and totally constrained models is discussed in more detail in Senior [19] and Wilson [33]

distance becomes less of a barrier to movement over time, the power is a more appropriate function with which to model flows. This is not to suggest that migrants necessarily perceive and react to distance as if it is logarithmically transformed, but that, of the three functions, the power function decreases the relative separation of origins and destinations, which corresponds with the increase in the level of mobility that occurred over the study period.

With respect to the spatial patterns of residuals, and their relationships to distance, it can be said again that overall, distance is a significant explanatory variable. However, the discussion of results above indicated clearly that unique events (the economic boom in Alberta) and unique provinces (the Quebec language barrier) can distort severely the relationship between distance and migration. The relation is strong, but certainly not inviolable—a point that Hagerstrand made forcefully and explicitly, some thirty years ago [7].

Perhaps, intuitively, it is reasonable to suggest that the role of distance in migration is decreasing in importance as mobility increases. For example, interregional migrations usually involve a severing of social ties and a change of environment, regardless of whether they are 1000 or 2000 miles in length. Nevertheless, it can be said that two distinct patterns exist during the study period: longer term, regional flows; and shorter term, national flows. Superimposed on the strong regional flows that exist within western and Atlantic Canada are a series of specific, sometimes temporary, long distance flows focusing on major areas of economic development. The importance of these areas varies over time; for example, in the 1970s, when the relative growth of British Columbia and Alberta had a national impact on migrant flow patterns.

As in most migration systems, there is a pattern of shorter regional flows that appears to be relatively constant. However, the nature of Canada's spatial units, and the concentration of development in the industrialized core, seems to prompt the occasional undertaking of large-scale long distance, interregional moves to newly developing areas even though they tend to be less stable economically over time. Indeed, the unique period of the seventies, characterized by rapid development in Alberta, confirms the particular willingness of large numbers of Canadians to overcome the "barriers of distance" to take advantage of job-related opportunities whenever and wherever they arise.

In the modelling strategy outlined in this paper, the size of a predicted migration stream is a function of origin and destination size effects, and of distance and deterrence function effects. Many migration researchers, in Canada and elsewhere, are interested in the role of other variables in the migration process; for example, employment

opportunities, income levels, educational attainment, language barriers, climate, and so on, see [22]. This raises the obvious question of whether models of the type described herein can be reformulated in order to take into account the effects of such variables; that is, entropy maximizing models with links to additional explanatory variables.

It was pointed out in an earlier paper [19] that the method of minimum information provides an avenue for the incorporation of additional prior information in constrained, gravity-type, interaction models. In this approach to interaction modelling, it is possible to define prior probabilities with respect to characteristics of variables measured between zones (such as previously observed movements or language barriers), or with respect to the zones themselves (unemployment levels, demographic characteristics, and so on). This represents an interesting and potentially valuable approach to migration modelling, but one that seldom has been tested empirically. The limited number of results to date [2;16;24;31] do suggest, however, that this is a viable avenue for continued investigation.

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