

# **Quality of Life in Southern Ontario <sup>(1)</sup>**

**Dimitrios A. Giannias**

**Department of Economics**

**University of Crete**

**Athens, Greece**

This paper applies a structural approach to hedonic equilibrium models to derive a quality of life ranking of six cities in Southern Ontario, Canada, namely, Guelph, Kitchener, London, Sarnia, St. Catharines, and Windsor. Unlike previous work in the area (for example, Gyourko and Racy (1991), Blomquist et al. (1985, 1988), Roback (1982, 1988), and Rosen (1979)) the method employed in this paper allows researchers to investigate how ranking will be affected by changes in the distribution of housing characteristics and/or the distribution of local or city amenities.

The previous work in this area defines the quality of life index to be a linear function of local and/or city amenities and uses that index to rank urban areas. The contention is that the well being of economic agents depends (among other factors) on neighbourhood and city characteristics. The weights assigned to these amenities are linear functions of their implicit prices from the housing and/or labour market. To derive these weights, the features of the hedonic housing price and/or wage functions are empirically approximated using fitting criteria. This provides the flexibility of letting the data determine the price and/or wage equations at the cost of not being able to test whether the assumed functional forms are consistent among themselves and with the underlying economic structure.

This method, unlike the one followed in this paper, does not provide the equilibrium hedonic price and/or wage equations and it cannot predict the changes in the implicit prices of amenities that are implied by changes in exogenous parameters. The latter implies, for example, that a researcher would not be able to find how a ranking of urban areas is affected by changes in the mean of the air quality distribution.

The closed-form approach adopted by this paper was first pursued by Epple (1984, 1987), who acknowledged that a series of articles by Tinbergen (1959), contributed to his work. This approach makes prior assumptions about the characteristics of the economic agents interacting to form the hedonic equilibrium, such as assumptions about the functional form of

the utility function, and uses them to derive and estimate the form of the equilibrium hedonic function.

Rosen (1974) examined ways to extract information on the underlying preferences and technologies from the hedonic equation for a situation with only one product characteristic. Even for this simplified case, the calculations required were quite complex. The complexity led Rosen to propose a methodology for estimating the demands and supplies of characteristics in a second stage rather than using the hedonic equation directly.

Epple (1984, 1987) took a differentiated product with an arbitrary number of characteristics and assumed the following:

- a quadratic utility function in each of the characteristics but additively separable between them;
- all consumers have the same utility function except for a differing taste parameter that was normally distributed with a diagonal covariance matrix; and,
- supply of the differentiated product is exogenous and also distributed normally with no covariance.

Giannias (1989) relaxes these assumptions by having the utility function depend on an index of the characteristics instead of on the characteristics themselves and he modified the model to examine Houston residents' willingness to pay for better air quality. The ability of the closed form approach in applied work is restricted by prior assumptions, for example, particular forms of quadratic utility functions as in Epple (1984, 1987) and Giannias (1989). This paper uses the closed form approach to estimate quality of life variations in Southern Ontario assuming that the quality of life is a scalar index and a linear function of housing, neighbourhood, and city characteristics. Imposing these prior restrictions helps provide the additional theoretical information that is essential in analyzing the housing market, estimating the quality of life index equation, identifying testable implications of the model, and providing a quality of life based ranking of urban areas. The estimation results are used to study the effects of changes in exogenous parameters on the quality of life rankings of Guelph, Kitchener, London, St. Catharines, Sarnia, and Windsor.

The first section introduces the theoretical model. This model assumes that the income distribution and the supply for housing characteristics are exogenous and that consumers use the services of only one house. The model is estimated and tested in the second section and quality of life rankings are discussed in the third section. Concluding remarks are given in the last section.

## The Theoretical Model

A competitive economy in which individuals consume one unit of a differentiated good and the numeraire good,  $x$ , is considered. Consumer preferences are described by a utility function  $U(h,x;a)$ , where  $h$  is the quality of the differentiated good (a scalar) and  $a$  is a  $(1 \times n)$  vector of utility parameters that differentiates consumers. The utility function is assumed to be a quadratic of the following form <sup>(2)</sup>:

$$U(h,x;a) = k_0 + (k_1 + k_2 a)h + 0.5k_3 h^2 + k_4 xh$$

Where  $k_i$  is a utility parameter ( $i = 0, 1, 3, 4$ ),  $k_2$  is a  $(1 \times n)$  vector of utility parameters, and  $a$  is the transpose of  $a$ . The  $[1 \times (n+1)]$  vector  $z = [a \ I]$  is assumed to follow an exogenous normal distribution, where  $I$  is the annual consumer income.

The differentiated good is described by the vector of attributes that describes a consumer's environment, namely, housing, neighbourhood, and city characteristics. These characteristics specify the quality of that composite commodity-environment which will be referred to as quality of life. There is a quality of life index that corresponds to each housing-locational choice. In equilibrium, there is a quality of life distribution for each city. A comparison of these distributions can provide a quality of life based ranking of urban areas.

A consumer solves the following optimization problem:

$$\max_{h,x} U(h, x; a)$$

subject to:

$$I = 12P(h) + 365x$$

Where  $P(h)$  is the equilibrium price equation (it gives the gross monthly rent of a house as a function of the quality index that corresponds to that house), 12 is the number of months in a year, 365 is the number of days in a year,  $I$  is the consumer's annual income, and  $x$  is the number of units of the numeraire good that are available to the consumer daily. When consumers choose housing, they consider the whole package of characteristics  $v$ . This vector of characteristics  $v$  is mapped into an index that defines the quality of life that corresponds to a housing choice. Since utility depends on  $h$ , the rental equilibrium price equation is a function of  $h$ , the parameters of which will depend on the characteristics of the distributions of consumer income and characteristics and be determined from the equilibrium. The quality of life is assumed to be a scalar and linear in  $v$ , that is,

$$h = e_0 v_1 + e_1 v_2 + \dots + e_{m-1} v_m$$

Where,  $e = [e_0 \dots e_{m-1}]$  is a vector of parameters, and  $v = [v_1 \dots v_m]$  is the vector of the characteristics of the differentiated good. The supply for  $v$  is assumed to follow an exogenous multinormal distribution.

Solving the utility maximization problem to obtain the demand for  $h$  and substituting it into the equilibrium condition (aggregate demand for  $h$  equal to aggregate supply for  $h$ ) gives, for all  $h$ , the equilibrium price equation for the economy described above <sup>(3)</sup>:

$$P(h) = q_0 + q_1 h$$

$$\text{where } q_1 = 365 (k_3 + A)/(24 k_4)$$

$$q_0 = 365 [k_1 + r m(z) - (2 k_4 q_1 - k_3) m(h)]/(12 k_4)$$

$$m(h) = m(v_1) + e_1 m(v_2) + \dots + e_{m-1} m(v_m)$$

$$V(h) = e V(v) e$$

$$A = [r V(z) r / V(h)]^{0.5}$$

$$r = [k_2 (k_4/365)]$$

Where  $m(t)$  is the mean of a variable  $t$ , for all  $t$ , and  $V(s)$  is the variance-covariance matrix of a vector of variables  $s$ , for all  $s$ .

The above equations are the result of the equilibrium of the economy and they specify the relationships between the price equation parameters and the structural and exogenous parameters of the model, namely, the utility parameters, the mean and the variance of income and consumer characteristic distributions, and the parameters of the quality index equation. The distributional assumptions about  $v$  and the assumption that the quality index equation is linear in the vector  $v$  imply that the distribution of prices in equilibrium is normal.

#### Estimation of the Model

To estimate the quality of life in Southern Ontario using the above model, the elements of the vector  $v$  that describes the environment of a consumer are assumed to be the following:

$v_1$  = the number of rooms of a house;

$v_2$  = the age of a house (measured in years);

$v_3$  = the local air quality index;

$v_4$  = the mean annual temperature of a city (measured in degrees cel-sius);

$v_5$  = the crime rate variable (the offense rate of a city per 100,000 of population).

The air quality variable,  $v_3$ , equals the inverse of the air pollution variable total suspended particulate matter (measured in microgram per cubic meter). Without loss of generality the quality of life is normalized by setting  $e_0$  is equal to 1, and dummy intercepts were included in the quality index equation so that other factors not included in  $v$ , are taken into account. Thus, the quality of life is given by:

$$h = \sum_c \epsilon_c D_c + e_0 v_2 + \dots + e_4 v_5$$

Where,  $\epsilon_c$  is a parameter,  $D_c$  is a dummy variable for all  $c$  ( $c$  is equal to Kitchener, London, St. Catharines, Sarnia or Windsor).

Given the above, the results of the previous section and assuming an additive error term, one can substitute the quality of life index equation into the equilibrium price equation to obtain:

$$P = b_0 + \sum_c \beta_c D_c + b_1 v_1 + \dots + b_5 v_5 + u \tag{1}$$

Where  $u$  is the econometric error,  $b_i$  and  $\epsilon_c$  are parameters to be estimated,  $b_j = q_1 e_{j-1}$  for  $j = 1, 2, 3, 4, 5$ , and  $\epsilon_c = q_1 \epsilon_c$  for all  $c$ . Note that from our normalization we have  $b_1 = q_1$ , which implies:

$$\varepsilon_c = \frac{\beta_c}{b_1} \quad \text{for all } c \quad (3)$$

and

$$e_{j-1} = \frac{b_j}{b_1} \quad \text{for } j = 1, \dots, 5 \quad (2)$$

Given estimates of the price equation parameters, (2) and (3) can be used to obtain the parameter estimates of the quality of life equation.

The model is estimated using census tract data for Guelph, Kitchener, London, St. Catharines, Sarnia, and Windsor that are obtained from 1981 Canada Census. These data were matched with the air quality measurements and city wide characteristics regarding temperature, precipitation, and crime rate. To obtain data concerning the annual arithmetic mean of total suspended particulate, all the monitoring stations in these six cities (given their addresses) were located according to census tract. The readings for these census tracts were used to represent pollution readings in adjacent census tracts since most cities contain a limited number of monitoring stations. If a census tract was adjacent to more than one census tract containing a monitoring station, then the average of the readings were used. These readings were then inverted so that the figures reflect air quality instead of air pollution. Note that within this framework the use of census tract data is justified because the equilibrium price equation is linear in housing, neighbourhood, and city characteristics.

The equilibrium price equation is estimated by ordinary least squares and the results are given in Table 1. Table 1, equations (2) and (3) imply that the quality of life equation is the following:

$$h = 0.038D_{KIT} - 0.270D_{LON} - 0.294D_{ST\ CAT} - 0.848D_{SAR} - 0.691D_{WIN} + v_1 - 0.046v_2 + 212.08v_3 + 0.107v_4 - 0.038v_5 \quad (4)$$

To see if the model makes a significant contribution to explaining the data, the hypothesis that all the coefficients of equation (1) are equal to zero is tested. This hypothesis is rejected at the 1% level of significance.

The analysis of the previous section implied that the equilibrium price distribution is normal. To investigate the internal consistency of the theory (given additive error terms) the joint

normality of gross rental prices and of the characteristics that describe the environment of a consumer was tested. To be more specific, the null hypothesis that the error term of equation (1) is normally distributed was tested. An omnibus test using  $X^2(C_1) + X^2(C_2)$  provides evidence in favour of the null hypothesis, where  $X^2(C_1)$  and  $X^2(C_2)$  are standardized normal equivalents to the sample skewness,  $C_1$ , and kurtosis,  $C_2$ .

**TABLE 1 The Price Equation**

Variable	Coefficient	Standard Error	T-Statistic
Intercept	37.438	100.940	0.371
D <sub>KIT</sub>	1.400	0.695	2.013
D <sub>LON</sub>	-10.086	-13.776	-0.732
D <sub>ST CAT</sub>	-10.960	-9.558	-1.147
D <sub>SAR</sub>	-31.674	-15.709	-2.016
D <sub>WIN</sub>	-25.780	-14.958	-1.723
v <sub>1</sub>	37.333	10.772	3.466
v <sub>2</sub>	-1.717	-0.704	-2.437
v <sub>3</sub>	7917.521	3744.718	2.114
v <sub>4</sub>	4.007	2.012	1.992
v <sub>5</sub>	-1.405	-4.492	-0.313

Note: Number of observation is 84 and  $R^2=0.73$

This normality test implies that the price equation, given  $v$ , is linear in  $v$ . <sup>(4)</sup>

#### Quality of Life Based Rankings of Six Cities in Southern Ontario

In equilibrium, each of the cities that are considered in this study, namely, Guelph, Kitchener, London, Sarnia, St. Catharines, and Windsor, provides a different quality of life distribution to its residents. The results of the previous section can be used to obtain the mean of this distribution,  $h_1$ , for each city. The mean for each city is given in Table 2. To obtain the  $h_1$  value of a city, the city's mean of  $v_i$ , for all  $i$  ( $i = 1, 2, \dots, 5$ ), is substituted into equation (4).

Another quality of life index,  $h_2$ , is obtained when the housing characteristics  $v_1$  and  $v_2$  are held constant across cities. To obtain the  $h_2$  value of a city (given in Table 2) the mean, over the six cities, of  $v_1$  and  $v_2$  and the city's mean of  $v_i$  ( $i = 3, 4, 5$ ) is substituted into equation (4).

In Table 2, the  $h_1$  and  $h_2$  values are also scaled from 0 to 100. <sup>(5)</sup>

**TABLE 2 Quality of Life Based Rankings**

	Rank	$h_1$ ( $H_1$ )	Rank	$h_2$ ( $H_2$ )
Guelph	1	8.99 (100.00)	1	8.45 (100.00)
Kitchener	2	8.26 (39.22)	2	8.38 (91.49)
London	3	7.86 (6.55)	3	8.37 (89.08)
St. Catharines	4	7.85 (6.13)	4	8.17 (63.81)
Sarnia	6	7.78 (0.00)	5	7.90 (27.12)
Windsor	5	7.83 (4.01)	6	7.69 (0.00)

Note:  $H_1$  and  $H_2$  are the  $h_1$  and  $h_2$  quality of life values scaled from 0 to 100, respectively.

Table 2 shows that Guelph scores higher than all other cities of Southern Ontario according to both criteria. However, the  $h_1$  scaled values,  $H_1$ , show that 5 cities score below 40, and the  $h_2$  scaled values,  $H_2$ , show that 5 of them score over 25. Thus, the relative differences from the quality of life indices of Guelph are greater in the case of  $H_1$  and smaller in the case of  $H_2$ , which indicates that the housing characteristics of Guelph are significantly better than those of the other 5 cities of Southern Ontario.

Our analysis also shows that Windsor is ranked fifth by the  $h_1$ -based ranking and Sarnia is ranked sixth. However, on the  $h_2$ -based ranking their position are sixth and fifth respectively. This shows that the housing characteristics are relatively better in Windsor, while the neighbourhood, and city characteristics are relatively better in Sarnia.

In addition, the structural analysis that is introduced in this paper allows us to estimate the effects of changes in exogenous parameters. Tables 3 and 4 give the impact, *ceteris paribus*,



of a 10% increase  $v_i$  on the  $h_1$ - and  $h_2$ -based rankings of each city given in Table 2. To be more specific:

Table 3 gives the impact of a 10% increase in  $v_i$ , *ceteris paribus*, on the  $h_1$ -based ranking of Table 2. For example, a 10% increase in  $v_3$  will make Sarnia's  $h_1$  quality of life index equal to 8.16 which is greater than that of Windsor (7.83), St. Catharines (7.85), and London (7.86). That is, Sarnia was ranked sixth before the 10% increase in air quality and third after it, which is an improvement of its relative ranking by three positions.

**TABLE 3 Impact of a 10% Increase in  $v_i$  on the  $H_1$ -Based Ranking**

	$R_c$					$\sqrt{R_c}$
	$v_1=10\%$	$v_2=10\%$	$v_3=10\%$	$v_4=10\%$	$v_5=10\%$	
Guelph	0	0	0	0	0	0
Kitchener	0	0	0	0	0	0
London	+1	-3	0	0	-2	6
St. Catharines	+2	-2	+1	+1	-1	7
Sarnia	+4	0	+3	+3	0	10
Windsor	+3	-1	+2	+2	-1	9
$\sum R_c$	10	6	6	6	4	--

Note: 1.  $R_c$  is the  $c$ -city's change in the  $H_1$ - based ranking of Table 2 when the  $v_i$  of the  $c$ -city increases 10%, *ceteris paribus*.

2.  $\sum R_c$  is the "vertical" sum of the absolute values of  $R_c$ .

3.  $\sqrt{R_c}$  is the "horizontal" sum of the absolute values of  $R_c$ .

4.  $c$  is Guelph, Kitchener, London, St. Catharines, Sarnia or Windsor.

**TABLE 4 Impact of a 10% Increase in  $v_i$  on the  $H_2$ -Based Ranking**

	$R_c$			$\sqrt{R_c}$
	$v_3=10\%$	$v_4=10\%$	$v_5=10\%$	
Guelph	0	0	0	0
Kitchener	+1	+1	-1	3

London	+2	+1	0	3
St. Catharines	+3	0	0	3
Sarnia	+1	0	0	1
Windsor	+1	0	0	1
${}_cR_c$	8	2	1	--

Note: 1.  $R_c$  is the  $c$ -city's change in the  $H_1$ - based ranking of Table 2 when the  $v_i$  of the  $c$ -city increases 10%, *ceteris paribus*.

2.  ${}_cR_c$  is the "vertical" sum of the absolute values of  $R_c$ .

3.  ${}_vR_c$  is the "horizontal" sum of the absolute values of  $R_c$ .

4.  $c$  is Guelph, Kitchener, London, St. Catharines, Sarnia or Windsor.

Table 4 gives the impact of a 10% increase in  $v_i$ , *ceteris paribus*, on the  $h_2$ -based ranking of Table 2. For example, a 10% increase in  $v_4$  will make London's  $h_2$  quality of life index equal to 8.44 which is greater than that of Kitchener (8.38). That is, London was ranked third before the 10% increase in air quality and second after it, which is an improvement of its relative ranking by one position.

Guelph's first place rank in Table 2 is not affected by the changes in  $v_i$  examined in either Table 3 or 4. The city that is most affected by changes in  $v_i$  is Sarnia in Table 3, and Kitchener, London, and St. Catharines in Table 4. As a measure to identify which city is most affected by the examined  $v_i$  changes we can take, for each city, the horizontal sum of the absolute values of the changes in its position on the initial ranking,  ${}_vR_c$ ; the  ${}_vR_c$  values are given in the last column of Tables 3 and 4. This value equals 10 for Sarnia in Table 3, and 3 for Kitchener, London, and St. Catharines in Table 4.

Similarly, the vertical sum of the  $R_c$  values in Tables 3 and 4, given by  ${}_cR_c$ , shows which of the examined changes in  $v_i$  affects the quality of life rankings in Table 2 most. From Table 3 we see that the changes in  $v_1$  affect more the initial  $h_1$ -based ranking of Table 2, while from Table 4 we see that changes in  $v_3$  seem to affect the  $h_2$ -based ranking of Table 2 more than the changes in  $v_4$ , and  $v_5$ .  ${}_cR_c$  equals 10 for  $v_1$  in Table 3 and 8 for  $v_3$  in Table 4.

### Conclusion

This paper presented a methodology to analyze quality of life based on a hedonic general equilibrium model. The structural analysis employed allows computation of the effects of changes in exogenous parameters.

The empirical results show that Guelph is ranked highest in both  $h_1$ - and  $h_2$ -based rankings. The results also show that the quality of life differences and rankings are affected by differences in the distribution of housing characteristics and that the variation in the values of the quality of life indices diminishes when the housing characteristics are held constant across cities.

#### References

Blomquist, G., M. Berger and J. Hohen. 1988. "New Estimates of Quality of Life in Urban Areas". *The American Economic Review*, 78: 89-107.

Blomquist, G., M. Berger and W. Waldner. 1985. "Quality of Life: Ranking Urban Areas Using a Wage and Rent Based Index". University of Kentucky, Department of Economics, working papers, no. E-85-85.

D'Agostino, R.B. and E.S. Pearson. 1973. "Tests for Departures from Normality. Empirical Results for the Distribution of  $b_2$  and  $b_1$ ". *Biometrika*, 60: 613-622.

Epple, D. 1984. "Closed Form Solutions to a Class of Hedonic Equilibrium Models". Carnegie-Mellon University, mimeo.

\_\_\_\_\_. 1987. "Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products". *Journal of Political Economy*, 95: 58-80.

Giannias, D.A. 1989. "Consumer Benefit from Air Quality Improvements". *Applied Economics*, 21: 1099-1108.

Gyourko, J. and J. Tracy. 1991. "The Structure of Local Public Finance and the Quality of Life". *Journal of Political Economy*, 99: 774-805.

Roback, J. 1982. "Wages, Rents, and the Quality of Life". *Journal of Political Economy*, 90: 1257-1278.

\_\_\_\_\_. 1988. "Wages, Rents and Amenities: Differences Among Workers and Regions". *Economic Inquiry*, January: 23-41.

Rosen, S.R. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition". *Journal of Political Economy*, 82: 34-55.

\_\_\_\_\_. 1979. "Wages-Based Indexes of Urban Quality of Life", in P. Mieszkowski and M. Straszheim (eds.). *Current Issues in Urban Economics*. Baltimore: Johns Hopkins University Press.

Tinbergen, J. 1959. "On the Theory of Income Distribution", in Klassen, Koyck, and Witteveen (eds.). *Selected Papers of Jan Tinbergen*. North-Holland.

#### Endnotes

1. The research was funded by the Jerome Levy Economics Institute of Bard College and The European Centre for Economic and Technical Studies, EKOM.

2. The structural approach to hedonic equilibrium models requires prior assumptions about the functional form of the utility function. There are two alternative functional forms that can be used: one was introduced by Epple (1984, 1987) and the other by Giannias (1989). In this paper we use the second for reasons explained in the introduction.

3. The methodology for obtaining the equilibrium price equation is given in Giannias (1989). The general strategy of the proof was introduced by Tinbergen (1959) and extended by Epple (1984) and Giannias (1989).

4. See D'Agostino and Pearson (1973). For this test the following composite test statistic is used:  $(N/6) (C_1)^2 + (N/24) (C_2 - 3)^2$ , where N is the number of observations. The statistic is distributed as a  $\chi^2$  with 2 degrees of freedom.

5. A series  $Y$ ,  $Y = m_i(h), SD_i(h), QOL_i$ , is scaled from 0 to 100 using the following transformation:  $Y^* = 100 (Y - Y_{\min}) / (Y_{\max} - Y_{\min})$ , where  $Y^*$  is the transformed index,  $Y_{\min}$  is the minimum value of  $Y$ , and  $Y_{\max}$  is the maximum value of  $Y$ .

Contact the journal at: [dwserve@nb.sympatico.ca](mailto:dwserve@nb.sympatico.ca)