

## **Technical Change and Regional Wage Rates**

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While most regional scientists feel the neoclassical approach is too limited, it nonetheless is implicit in much of the conventional wisdom and approach (Leven 1985) so that "...both the mechanisms for developing peripheral areas, and the variables used for measuring the success achieved, have essentially been borrowed from neoclassical economics" (Stöhr 1982: 71). To an extent perhaps unusual in other countries, Canadian policy discussions and even the academic literature on regional problems are dominated by the neoclassical paradigm.

A conventional treatment of the role of technological diffusion in a regional context is to attribute low wages in a region to slowness to adopt new production techniques (Batra and Scully 1972; Economic Council of Canada 1977, 1980; Macdonald Royal Commission 1985; Mansell and Copithorne 1986; Pereira and Seabra 1994). The link assumed is that management's failure to adopt new techniques in some regions is one cause of low wages. However, this is not, in itself, an explanation because it implies that entrepreneurial ability is an immobile resource. Otherwise, entrepreneurs from other regions should move into the technologically backward, low wage region (Bradfield 1976).

Moreover, the analysis of technical change is often confusing because the terms technological and technical change are used for three different phenomena: changes in the capital intensity within a given technique, changes to a new technique, and changes in the general knowledge or efficiency base (although not necessarily in the production technique). Mansell and Copithorne (1986: 16) state that it "is well known that the amount of capital per worker is an important determinant of per worker output and earnings". This is true within a given production function but it cannot be assumed that an increase in the capital/labour ratio will have this effect on labour earnings when it occurs because of a shift to a new production technique.

Moreover, there is often a failure to distinguish between technological change (an exogenous change in factors affecting general efficiency levels) and technical change (a change in the

production technique used by individual firms). The latter is often what is intended by technological change since it is implied that lagging regions need to adopt more capital-intensive (more modern?<sup>(1)</sup>) techniques (Casetti and Tanaka 1992; Macdonald 1985) in order to raise wages. In a more general context, it has been stated that "technical progress and capital deepening would produce rising real wages..." (Osberg and Phipps 1992: 14). Others argue that higher wages have led to increased capital intensity (Moriarty 1992). Rivera-Batiz and Xie (1993: 338) cite "empirical investigations that have shown that technological change is a significant determinant of secular increases in per capita income" but this need not mean that wages rise since the growth in incomes could reflect falling wages but still larger increases in profits.

In addition, a strong faith in economies of scale, combined with studies such as Haldi and Whitcomb (1967), leads many to conclude that technical change will be "biased toward capital intensive techniques" (McDonough 1992). Cekota (1988) estimates that the capital-labour ratio increases by more than 70 percent as a result of technological change which implies a significant degree of substitution of capital for labour.<sup>(2)</sup>

This paper analyzes the relationship between the regional diffusion of technical innovation and wage rates by questioning the neoclassical assumption that a change in production technique increases wages, *ceteris paribus*. Using a competitive model, it is shown that the adoption of a new production technique does not necessarily increase wages since there are contradictory effects on wages - the change in the production function parameter changes the equilibrium capital/labour ratio and the marginal product of labour for a given capital/labour ratio in opposite directions. The model suggests that technical change, by itself, may actually lower wages! It is clarified that technological change, not technical change, increases wages.

The first section sets up the model and demonstrates the conditions under which the adoption of new techniques of production may lower wages. The second section shows how technological change raises wages. The third section discusses the policy implications and provides a summary and conclusion. The appendix generalizes the results by employing a CES production function.

### **The Effect of Technical Change on Wages**

Following Bradfield (1976), let us assume an industry which is competitive in both its factor and product markets, producing a product with the well-behaved, albeit restrictive,<sup>(3)</sup> Cobb-Douglas production function:

$$X = P_q Q = P_q A (CK)^\alpha (BL)^{1-\alpha} \quad (1)$$

where  $X$  is the value of the physical output,  $Q$ ,  $P_q$  is the price of output,  $A$ ,  $B$ , and  $C$  are the neutral, labour-specific, and capital-specific efficiency co-efficients, respectively,  $K$  and  $L$  are the inputs of capital and labour, respectively, and is the production function parameter.

In terms of the distinction between technological change and technical change, a change in represents technical change, a change in the technique of production. A change in  $A$ ,  $B$ , or  $C$  would represent a change in the knowledge base or factor efficiencies, that is, technological change. Of course, the change in technique presumably also reflects the discovery of a new way of producing  $Q$ , and therefore also picks up some of the effects of technological change.

In a competitive industry, each firm can be assumed to have exhausted all economies of scale and to be working in the range of output where the production function is homogeneous of degree one. For the competitive firm,  $P_q$  is given. This may also be true for some or all of the regional submarkets of the national industry and even for the national industry itself, if international markets are important to it. On the other hand, the national market and large regions may face a downward sloping demand curve for their output. For them,  $P_q$  would be a function of the level of output. The efficiency co-efficients,  $A$ ,  $B$ , and  $C$ , are assumed exogenous to both the firm and the industry.

Equilibrium in the capital market requires that capital be hired until the marginal revenue product of capital is equal to its cost:

$$r P_k = \frac{\partial X}{\partial K} = \alpha P_q A C^\alpha B^{1-\alpha} k^{\alpha-1} \quad (2)$$

where  $r$  is the rate of interest (the cost of financial capital),  $P_k$  is the cost of a unit of physical capital, and  $k$  is the capital/labour ratio,  $k = K/L$ .

Equation (2) can be re-arranged to determine the equilibrium capital/labour ratio:

$$k_e = (\alpha P_q A C^\alpha)^{\frac{1}{1-\alpha}} B^{1-\alpha} (r P_k)^{\frac{1}{\alpha-1}} \quad (3)$$

Similarly, labour market equilibrium means that the wage is equal to the marginal product of labour:

$$W_e = \frac{\partial X}{\partial L} = (1-\alpha) P_q A C^\alpha B^{1-\alpha} k_e^\alpha \quad (4)$$

Substituting from equation (3) for the capital/labour ratio in (4), the equilibrium wage rate can be expressed as

$$W_e = (1-\alpha)(\alpha^\alpha P_q A C^\alpha)^{\frac{1}{1-\alpha}} B (r P_k)^{\frac{\alpha}{1-\alpha}} \quad (5)$$

Taking differentials, noting that  $d \log V = V^* = V/V$ , from equation (5), we can express  $W_e^*$ , the rate of change in the equilibrium wage rate, as

$$\begin{aligned} W_e^* = & (1-\alpha)^* + \frac{1}{(1-\alpha)} [\alpha \alpha^* + P_q^* + A^* + \alpha (C^* - r^* - P_k^*)] \\ & + B^* + [d \frac{\alpha}{1-\alpha}] \log \alpha + [d \frac{\alpha}{1-\alpha}] \log (\frac{C}{r P_k}) \\ & + [d \frac{1}{1-\alpha}] \log (P_q A) \end{aligned} \quad (6)$$

Since

$$d \frac{\alpha}{1-\alpha} = d \frac{1}{1-\alpha} = d \frac{\alpha}{(1-\alpha)^2} = -(\frac{1}{1-\alpha}) (1-\alpha)^*$$

the rate of change in the equilibrium wage can be expressed as

$$\begin{aligned} W_e^* = & (1-\alpha)^* + \frac{1}{(1-\alpha)} [\alpha \alpha^* + P_q^* + A^* + \alpha (C^* - r^* - P_k^*)] \\ & + B^* - [\frac{(1-\alpha)^*}{(1-\alpha)}] \log (\alpha P_q A \frac{C}{r P_k}) \end{aligned} \quad (6')$$

Equation (6') seems to suggest that a change in technique, such as a change in  $\alpha$ , will directly affect the equilibrium wage rate,  $W_e$ . However, the effects of  $(1-\alpha)^*$  and of  $*/(1-\alpha)$  in equation (6) are not only opposite but equal. The algebra of this is relatively straightforward. The expression  $(1-\alpha)$  changes when  $\alpha$  changes:

$$\Delta(1-\alpha) = -\Delta\alpha \quad (7)$$

and therefore

$$\frac{\Delta(1-\alpha)}{(1-\alpha)} = [-\frac{\Delta\alpha}{(1-\alpha)}] \frac{\alpha}{\alpha} \quad (7')$$

or

$$(1-\alpha)^* = -\frac{\alpha^* \alpha}{(1-\alpha)} \quad [Q.E.D.] \quad (7'')$$

Thus  $(1-\alpha)^*$  cancels out the  $*/(1-)$  of equation (6) and it becomes

$$W_e^* = \frac{1}{(1-\alpha)} [P_q^* + A^* + \alpha(C^* - r^* - P_k^*)] + B^* - \left[ \frac{(1-\alpha)^*}{(1-\alpha)} \right] \log\left(\alpha P_q A \frac{C}{r P_k}\right) \quad (8)$$

The sign of the last argument of equation (8) is impossible to evaluate since  $[(1-\alpha)^*/(1-)]$  is negative when technical change is capital-intensive ( $\alpha^* > 0$ ), and  $\log(\alpha P_q A C / (r P_k))$  will be positive or negative depending on which is larger, the numerator or the denominator, respectively.

Since some of the effect of the change in the production function parameter,  $\alpha$ , cancels out in equation (8) and the remaining effect is indeterminate, increases in the equilibrium wage are not guaranteed by changes in the production technique, such as changes in  $\alpha$ . Indeed, depending on the factor intensity of technical change and on the value of  $(\alpha P_q A C)$  relative to  $(r P_k)$ , the effect of technical change may be to lower equilibrium wages. Thus, there is no unequivocal hope for a region that technical change will raise the equilibrium wage rate.<sup>(4)</sup>

It can be seen from equation (8) that the equilibrium wage can also change if there is a change in an underlying exogenous variable: the price of output, the efficiency co-efficients, or the cost of capital. Changes in the efficiency co-efficients represent the effects of technological change -- changes in the knowledge base of the industry or society or in infrastructure, resources, or other factors.

How do we explain the result of equation (8) since it contradicts conventional (neoclassical) expectations that a new technique will help push up wages? Assume that the change in technique is capital-intensive, so that at existing prices, etc., the capital/labour ratio increases. In this case we can use equation (3) to express the change in the equilibrium capital/labour ratio:

$$k_e^* = \frac{\alpha^*}{(1-\alpha)} + d \frac{1}{1-\alpha} \log\left(\alpha P_q A \frac{C}{r P_k}\right) \quad (9)$$

ceteris paribus. The capital/labour ratio changes directly with the parameter  $\alpha$ , but by a greater amount ( $*/(1-\alpha) > *$ ). If there is a change in production technique which increases the capital-intensity of the production function ( $\alpha^* > 0$ ), there are offsetting effects on the equilibrium marginal product of labour. In the capital market, the increase in  $k_e$  requires an increase in the equilibrium capital/labour ratio, from equation (9). Ceteris paribus, this would increase the

marginal revenue product of labour and drive up the equilibrium wage. However, this effect is offset by the direct effect on the marginal product of labour from the decline in (1-) in equation (4).

Thus, the more capital-intensive technique generates offsetting effects on the equilibrium wage, as would be equally true of a new production technique which was relatively more labour intensive ( $\alpha < 0$ ). In either case, the change in the production function parameter does not directly affect the equilibrium wage rate. With either capital- and labour-intensive technical change, the effect on the wage rate is not direct (through the change in the production function parameter) but through the other variables of equation (8). But even this effect of technical change may be perverse, since it can lower wages as seen by the indeterminate argument of equation (8),  $-\left[\frac{(1-\alpha)^*}{(1-\alpha)}\right] \log\left(\frac{P_q A C}{r P_k}\right)$ . This possibility of technical change lowering wages is enhanced by the other arguments of equation (8) since at least one of these "exogenous" variables,  $P_q$ , must eventually be affected by technical change.

An improvement in production technique would lower the cost of production which would mean that prices must eventually fall (Neary 1981) by the cost savings, as the new, more efficient technique is adopted throughout the competitive industry. Therefore any change in the production function parameter, whether a capital- or labour-intensive change, will drive the price of the output down. But equation (8) shows that the equilibrium wage is directly related to the price of output. A fall in the price of output will lower wages. Moreover, equation (8) shows that the equilibrium wage will fall by more than the fall in product price, since (8) becomes

$$W^* = \frac{P_q^*}{(1-\alpha)} - \left[\frac{(1-\alpha)^*}{(1-\alpha)}\right] \log\left(\alpha P_q A \frac{C}{r P_k}\right), \quad (10)$$

ceteris paribus. Since  $(1-\alpha)^* < 1$ , the effect on wages is a multiple of the drop in prices. This would then exacerbate or offset the effect of the second argument of equation (10), depending on whether the second argument's effect is to lower or to raise wages, respectively. Thus, we must conclude that there is a strong possibility that technical change, the adoption of new production techniques, will directly and indirectly lower wages.

### **Wages and Technological Change**

Of course, changes in the other underlying variables will affect the level of wages. An increase in any of the efficiency co-efficients will have a direct effect on wages, although the magnitude of the effect will vary depending on whether the change is in the neutral, labour- or capital-

specific co-efficient, since each has a different weighting in equation (8). The percent increase in wages will be equal to the percent rise in labour's efficiency, a multiple of any neutral efficiency increase, and a fraction of that multiple for changes in capital's efficiency. However, these changes are not changes in the production technique, but rather in those elusive factors which affect efficiency levels. Thus, they represent technological change. Nonetheless, a move to a more capital-intensive technique may require a more highly skilled labour force (Betts 1989), thereby raising the value of  $B$ , labour's efficiency co-efficient, as technical change requires that the firm upgrade its labour force.

If it is argued that a cost-saving change in technique ( $\Delta$ ) must be accompanied by an increase in at least the neutral efficiency level ( $A^* > 0$ ) to generate an unequivocal lowering of costs, then the change in  $A$  would affect the wage rate. Nonetheless, its effect on the wage rate will be the same, regardless of whether the shift in technology were capital- or labour-intensive. From equation (8), if  $\Delta$  requires an  $A^*$ , it is the  $A^*$  (the improvement in efficiency level), not the  $\Delta$  (the change in production technique), which unequivocally raises wages, regardless of whether  $\Delta$  is positive or negative. However, the effect of  $A^*$  on wages will be offset by the fall in output price generated by the increased efficiency in the firm and industry, since equation (8) shows that  $A^*$  and  $P_Q \Delta$  have the same weighting.

If technological change has focussed on changes in the efficiency co-efficients, these affect wages directly and through the price of the product. Moreover, the increased productivity from technological and technical change may well affect the level of savings and therefore the interest rate  $r$ , or affect the cost of physical capital  $P_k$ , as production becomes more efficient in the capital goods industries. These changes would raise both the equilibrium wage (equation (8)) and the capital/labour ratio (equation (3)). Nonetheless, for a given industry these effects do not represent a change in production technique which generates a change in the production function parameter  $\alpha$ . Technical change in the production of good  $Q$  (that is, a change in its production technique) will not, ceteris paribus, unambiguously drive wages up in that industry. Technical change in other industries affecting the efficiency or price of inputs for industry  $Q$  will affect wages in  $Q$ , but this effect is at least partially offset by the long term fall in the price of  $Q$ .

We have shown that a change in production technique does not, in itself, guarantee higher wages, and that technological change will increase wages but the effect will be offset to some extent by price adjustments. The analysis here suggests that the latter effect is not inevitable. It is now time to discuss the policy implications of these results.

## **Policy Implications**

Changing the production function will not automatically raise wages because of the contradictory effects of a change in technique on the equilibrium capital/labour ratio and on the marginal revenue product of labour at any given capital/ labour ratio. In addition, there would be downward pressure on wages because of the effect on product price of a more efficient technique. Thus the failure to adopt the latest technique does not explain regional wage differentials under the conditions assumed. Indeed, it can be shown that the cause-effect relationship may be reversed -- low wages may provide a profit-maximizing incentive to delay the introduction of a new technique (Bradfield 1988).

The model does confirm a second part of conventional wisdom -- technological change is important for growth and wage levels. Nonetheless, the impact on wages is modified when the effect of technological change on prices is included. Thus technical change and technological change have different effects on wages. This distinction between the effects of technical and technological change carries distinct policy implications. Government should not think it can raise local wages by assisting industries to update their production techniques. If new techniques tend to be capital-intensive, the updating will lead to fewer workers employed but may not generate higher wages for those who remain. Thus, the intent of the policy would not be attained but the side-effects would exacerbate regional disparities in unemployment.

This is one more reason to question the efficacy of regional capital subsidies. They have been criticized in the past for encouraging firms to use an existing production technique more capital-intensively by raising the wage/rent ratio (Woodward 1975). If capital subsidies also encourage firms to shift to more capital-intensive techniques, either because of the effect of the subsidies on the wage/rent ratio or because they include explicit efforts to promote technical change, their effects on decreasing employment will be compounded.

Clearly, policy should reflect the importance of technological change for raising wages. Unfortunately, this is a less obvious, slower and perhaps more expensive, process than that of subsidising capital. It involves improving the general efficiency level (factors which affect the  $A$  of the equations) through activities such as infrastructure improvement, achieving agglomeration economies where they exist, changing attitudes (of labour and management), and setting up new institutional arrangements which allow firms to get raw material inputs on better terms.



Improving labour's efficiency ( $B$ ) opens a Pandora's box of policies affecting areas such as education, health, and workers' attitudes. Of all the policy areas, this is perhaps most startling, in the sense that these are the areas where government actions are likely to be most wrong-headed. Governments have been reducing their support for education and health. They have been getting tougher on the labour force in areas such as occupational health and safety standards, access to unemployment insurance, and unionisation, thereby affecting workers' attitudes.

Increasing the efficiency of the existing capital stock ( $C$ ) may involve improved maintenance and repair, changes in production line configurations, better training of management and labour in the use of capital, etc. At least some of these policy approaches have been attempted in the past. Unfortunately, despite decades of research on technological change, and its importance as the residual variable accounting for both growth and regional disparities (Economic Council of Canada 1977), we know little about the sources of technological change. Similarly, we have limited evidence of the efficacy of policies for technological change, and therefore limited academic agreement on the appropriate tack to take.

It might be argued that the model employed here suggests that even encouraging technological changes may not have the effects on wages normally assumed, because of their effects on product price. However, the low wage regions are often residual producers in national and international markets and are, essentially, price takers. Thus technological change in their industries may be driving prices down in any event. To help these regions accelerate the process of technological change is therefore to at least help them keep wages from falling. To the extent that these regions are able to achieve productivity growth independent of trends in their industries, wages can be raised.

Of course, we might also look at policies which would affect the other variables of the neoclassical model, product price ( $P_q$ ), and the cost of real and financial capital ( $P_k$  and  $r$ , respectively). The limited work (Dow 1987; Hughes 1992) in these areas probably reflects the implicit dominance of the neoclassical paradigm which assumes these markets to be perfectly competitive (and without friction), except perhaps for the cost of transportation (Macdonald 1985). Nonetheless, one might expect that imperfectly competitive markets for a firm's product as well as for its capital (and other) inputs would suggest policies to overcome the imperfections. These types of discussions tend to be found in the local development literature, for instance in discussions on the importance of institutions to keep local savings available for local enterprise. The neoclassical response is to assume that interference in the private market is inefficient because of the assumption that the market is perfectly competitive.

In summary, the dominance of the neoclassical paradigm, combined with a failure to maintain the distinction between technical and technological change, have led to considerable faith being placed on new production techniques as a means of improving the conditions of lagging regions. This paper shows that new techniques need not improve wages, could lower them, and, if capital-intensive, will increase unemployment. Therefore policy must focus on the more nebulous concept of technological change.

#### Appendix A

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#### Appendix A

We can generalize our production function, for instance to a Constant Elasticity of Substitution (CES) function, of which the Cobb-Douglas is a special case. Because it is additive rather than multiplicative, the CES is much more difficult to manipulate to get meaningful results, so the following may seem somewhat obtuse.

Assume a CES production function with the same variables as the Cobb-Douglas specification:

$$X = AP_{\phi} [\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1}{\rho}} \quad (A1)$$

There  $a$  and  $b$  are production function parameters, and  $\rho$  is the substitution parameter, with  $-1 < \rho < \infty$  and  $\sigma = 1/(1 + \rho)$  where  $\sigma$  is the elasticity of substitution. The equilibrium conditions in the factor markets (factor prices = marginal revenue products) generates:

$$W = \frac{\partial X}{\partial L} = AP_{\phi} b B^{-\rho} L^{-\rho-1} [\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1-\rho}{\rho}} \quad (A2)$$

$$rP_x = \frac{\partial X}{\partial K} = AP_{\phi} \alpha C^{-\rho} K^{-\rho-1} [\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1-\rho}{\rho}} \quad (A3)$$

From (3) we can substitute for  $[\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1-\rho}{\rho}}$  in equation (A2) which becomes

$$W = [AP_q b B^{-\rho} L^{-\rho-1}] [r P_k A^{-1} P_q^{-1} \alpha^{-1} C^\rho K^{\rho+1}] \quad (A4)$$

or

$$W = \left(\frac{b}{a}\right) r P_k \left(\frac{C}{B}\right)^\rho \left(\frac{K}{L}\right)^{\rho+1} \quad (A5)$$

In terms of the capital/labour ratio, equation (5) can be expressed as:

$$W = \left(\frac{b}{a}\right) r P_k \left(\frac{C}{B}\right)^\rho k^{\rho+1} \quad \text{where } k = \frac{K}{L} \quad (A5')$$

In log form, equation (5') becomes

$$\ln W = \ln b - \ln a + \ln r + \ln P_k + \rho \ln C - \rho \ln B + (\rho+1) \ln k \quad (A6)$$

Therefore, the rate of change in the wage rate can be expressed as

$$W^* = b^* - a^* + r^* + P_k^* + \Delta \rho \ln C + \rho C^* - \Delta \rho \ln B - \rho B^* + \Delta \rho \ln k + (\rho+1) k^* \quad (A7)$$

$$W^* = b^* - a^* + r^* + P_k^* + \rho C^* \rho B^* + (\rho+1) k^* + \Delta \rho (\ln C - \ln B + \ln k) \quad (A8)$$

$$W^* = b^* - a^* + r^* + P_k^* + \rho(C^* - B^*) + (\rho+1) k^* + \Delta \rho \ln \left(\frac{k C}{B}\right) \quad (A9)$$

To derive in terms of factor shares, multiply the numerator and denominator of equation (A3) by  $(AP_q)$  to get

$$\begin{aligned} r P_k &= a (AP_q)^{\rho+1} [a(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1}{\rho}} (AP_q C)^{-\rho} K^{-\rho-1} \\ &= a \{AP_q [a(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1}{\rho}}\}^{\rho+1} (AP_q C)^{-\rho} K^{-\rho-1} \\ &= a X^{\rho+1} (AP_q C)^{-\rho} K^{-\rho-1} \end{aligned} \quad (A10)$$

Multiply both sides by  $(r P_k)^{-1}$ ,

$$\begin{aligned} (r P_k)^{-\rho} &= a X^{\rho+1} (AP_q C)^{-\rho} K^{-\rho-1} (r P_k)^{-\rho-1} \\ &= a (AP_q C)^{-\rho} \pi_k^{-\rho-1} \end{aligned} \quad (A11)$$

Therefore

$$\pi_k = \left( \frac{rP_k}{AP_q C} \right)^{\frac{\rho}{\rho+1}} \alpha^{\frac{1}{\rho+1}} \quad (\text{A12})$$

Labour's share of output is  $WL/X = L$ . From equation (A2),

$$\pi_L = bB^{-\rho}L^{-\rho}[\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{-1} \quad (\text{A13})$$

$$\pi_L^{\frac{\rho+1}{\rho}} = b^{\frac{\rho+1}{\rho}} B^{-\rho-1}L^{-\rho-1} [\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1-\rho}{\rho}} \quad (\text{A14})$$

or

$$[\alpha(CK)^{-\rho} + b(BL)^{-\rho}]^{\frac{-1-\rho}{\rho}} = \pi_L^{\frac{\rho+1}{\rho}} b^{-\frac{\rho+1}{\rho}} B^{\rho+1}L^{\rho+1} \quad (\text{A15})$$

Therefore

$$W = b^{\frac{-1}{\rho}} AP_q B^{\frac{\rho+1}{\rho}} \pi_L^{\frac{\rho+1}{\rho}} \quad (\text{A17})$$

$$W = bAP_q B^{-\rho}L^{-\rho-1} \pi_L^{\frac{\rho+1}{\rho}} b^{\frac{-\rho-1}{\rho}} B^{\rho+1}L^{\rho+1} \quad (\text{A16})$$

In log form, equation (17) becomes

$$\ln W = \frac{-1}{\rho} \ln b + \ln A + \ln B + \ln P_q + \frac{\rho+1}{\rho} \ln \pi_L \quad (\text{A18})$$

Therefore

$$\begin{aligned} \frac{\Delta W}{W} &= \frac{-1}{\rho} \frac{\Delta b}{b} + \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta P_q}{P_q} \\ &+ \frac{\rho+1}{\rho} \frac{\Delta \pi_L}{\pi_L} + \frac{\Delta \rho}{\rho^2} \ln b - \frac{\Delta \rho}{\rho^2} \ln \pi_L \end{aligned} \quad (\text{A19})$$

or

$$\begin{aligned} \frac{\Delta W}{W} &= \frac{-1}{\rho} \frac{\Delta b}{b} + \frac{\Delta A}{A} + \frac{\Delta P_q}{P_q} + \frac{\rho+1}{\rho} \frac{\Delta \pi_L}{\pi_L} \\ &+ \frac{\Delta \rho}{\rho^2} [\ln b - \ln \pi_L] \end{aligned} \quad (\text{A20})$$

Since  $L = 1 - \kappa$

$$\Delta \pi_L = -\Delta \pi_K \quad (\text{A21})$$

$$\frac{\Delta \pi_L}{\pi_L} = \frac{-\Delta \pi_K}{1 - \pi_K} = \frac{-\Delta \pi_K}{\pi_K} \left( \frac{\pi_K}{\pi_L} \right) \quad (\text{A22})$$

that is

$$\pi_L^* = \left( \frac{-\pi_K}{\pi_L} \right) \pi_K^* \quad (\text{A23})$$

Therefore

$$\begin{aligned} W^* = & -\frac{1}{\rho} b^* + A^* + B^* + P_q^* + \left( \frac{\rho+1}{\rho} \right) \left( \frac{-\pi_K}{\pi_L} \right) \pi_K^* \\ & + \frac{1}{\rho} \rho^* [\ln b - \ln \pi_L] \end{aligned} \quad (\text{A24})$$

If  $L = b$ ,  $K = a$  where  $a + b = 1$  then

$$\begin{aligned} W^* = & \frac{-1}{\rho} b^* + \left( \frac{\rho+1}{\rho} \right) \left( \frac{-a}{b} \right) a^* + A^* + B^* \\ & + P_q^* + \frac{1}{\rho} \rho^* [\ln b - \ln b] \end{aligned} \quad (\text{A25})$$

$$b^* = \frac{-a}{b} a^*$$

But

therefore

$$W^* = \frac{-1}{\rho} b^* + \frac{\rho+1}{\rho} (b^*) + A^* + B^* + P_q^* \quad (\text{A26})$$

that is

$$W^* = b^* \text{ if } \pi_L = b, \pi_K = a, \quad (\text{A27})$$

and  $A$ ,  $B$ , and  $P_q$  are fixed.

To show how miserable life can be when wages are not expressed in terms of factor shares or the capital/labour ratio, express equation (A2) in logs:

$$\begin{aligned} \ln W = & \ln A + \ln P_q + \ln b - \rho \ln B - (\rho+1) \ln L \\ & - \frac{(1+\rho)}{\rho} \ln [\alpha (CK)^{-\rho} + b(EL)^{-\rho}] \end{aligned} \quad (\text{A28})$$

Expressed as the percentage increase in wages, equation (A28) becomes

$$\begin{aligned}
\frac{\Delta W}{W} = W^* &= \frac{\Delta A}{A} + \frac{\Delta P_q}{P_q} + \frac{\Delta b}{b} - \rho \frac{\Delta B}{B} - \ln B \Delta \rho - (\rho+1) \frac{\Delta L}{L} - \ln L \Delta \rho \\
&+ \frac{\Delta \rho}{\rho^2} \ln [a(CK)^{-\rho} + b(BL)^{-\rho}] - \frac{(1+\rho)}{\rho} \left[ \frac{\Delta a(CK)^{-\rho} + \Delta b(BL)^{-\rho}}{a(CK)^{-\rho} + b(BL)^{-\rho}} \right] \\
&- \frac{(1+\rho)}{\rho} \left[ \frac{a(CK)^{-\rho} [\Delta \rho (-\ln C - \ln K) - \rho (\frac{\Delta C}{C} + \frac{\Delta K}{K})]}{a(CK)^{-\rho} + b(BL)^{-\rho}} \right] \\
&- \frac{(1+\rho)}{\rho} \left[ \frac{b(BL)^{-\rho} (\Delta \rho (-\ln B - \ln L) - \rho (\frac{\Delta B}{B} + \frac{\Delta L}{L}))}{a(CK)^{-\rho} + b(BL)^{-\rho}} \right]
\end{aligned} \tag{A29}$$

Therefore, the rate of change in the share of output going to capital when only the production function parameters are changing can be expressed as

$$\pi_k^* = s\alpha^* \tag{A10}$$

Finally, to show how absolutely miserable the CES can make things, we can find the determinants of the equilibrium capital/labour ratio ( $k$ ) from equation (A3):

$$rP_x = AP_q a C^{-\rho} K^{-\rho-1} [a(CK)^{-\rho} + b(BL)^{-\rho}]^{-\frac{1-\rho}{\rho}} \tag{A3}$$

Therefore

$$\begin{aligned}
rP_x &= AP_q a C^{-\rho} (K^{-\rho})^{\frac{-\rho-1}{\rho}} [a(CK)^{-\rho} + b(BL)^{-\rho}]^{-\frac{1-\rho}{\rho}} \\
&= AP_q a C^{-\rho} (K^{\rho})^{\frac{-\rho-1}{\rho}} [a C^{-\rho} K^{-\rho} + b B^{-\rho} L^{-\rho}]^{-\frac{1-\rho}{\rho}} \\
&= AP_q a C^{-\rho} [a C^{-\rho} K^{-\rho} K^{\rho} + b B^{-\rho} L^{-\rho} K^{\rho}]^{-\frac{-\rho-1}{\rho}} \\
&= AP_q a C^{-\rho} [a C^{-\rho} + b B^{-\rho} k^{\rho}]^{-\frac{-\rho-1}{\rho}}
\end{aligned} \tag{A30}$$

or,

$$(rP_x)^{\frac{-\rho}{\rho+1}} = (AP_q a C^{-\rho})^{\frac{-\rho}{\rho+1}} [a C^{-\rho} + b B^{-\rho} k^{\rho}] \tag{A31}$$

Re-arranging terms, equation (A31) becomes

$$(AP_q P_x r a C^{-\rho})^{\frac{-\rho}{\rho+1}} = a C^{-\rho} + b B^{-\rho} k^{\rho} \tag{A32}$$

and



$$\begin{aligned}
k^p &= \frac{B^p}{b} \left( \frac{AP_q a}{P_x r C^p} \right)^{\frac{p}{p+1}} - \frac{a C^{-p} B^p}{b} \\
&= \frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p
\end{aligned} \tag{A33}$$

and therefore

$$k = \left[ \frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p \right]^{\frac{1}{p}} \tag{A34}$$

or

$$\ln k = \frac{1}{p} \ln \left[ \frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p \right] \tag{A35}$$

Assume everything is fixed except  $a$ ,  $b$ ,  $r$ ,  $C$ ,  $P_x$ ,  $P_q$ ,  $A$ ,  $B$ ,  $k$

$$\begin{aligned}
\frac{\Delta k}{k} &= -\frac{\Delta \rho}{\rho^2} \ln \left[ \frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p \right] \\
&+ \frac{1}{\rho} \frac{-\frac{\Delta b}{b^2} B^p \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{\Delta a}{b} \left( \frac{B}{C} \right)^p + \frac{a \Delta b}{b^2} \left( \frac{B}{C} \right)^p}{\frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p} \\
&+ \frac{1}{\rho} \frac{\frac{B^p \ln B}{b} \Delta \rho \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} + \frac{\Delta \rho}{(\rho+1)^2} \ln \left( \frac{a A P_q}{r C^p P_x} \right) + \frac{\rho}{\rho+1} \left( \frac{\Delta a}{a} - \Delta \rho \ln C \right)}{\frac{B^p}{b} \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p} \\
&+ \frac{1}{\rho} \frac{\frac{a}{b} \left( \frac{B}{C} \right)^p \ln \left( \frac{B}{C} \right) \cdot \Delta \rho}{\left( \frac{B^p}{b} \right) \left( \frac{a A P_q}{r C^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p}
\end{aligned} \tag{A36}$$

Expressed as rates of change, equation (A36) becomes

$$\begin{aligned}
k^* &= -\frac{\rho^*}{\rho} \ln \left[ \frac{B^p}{b} \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p \right] \\
&+ \frac{1}{\rho} \frac{-\frac{b^*}{b} B^* \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} - a^* \frac{a}{b} \left( \frac{B}{C} \right)^p + b^* \frac{a}{b} \left( \frac{B}{C} \right)^p}{\frac{B^p}{b} \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p} \\
&+ \frac{\frac{B^p \ln B}{b} \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} \rho^* + \frac{\rho^*}{(\rho+1)^2} \ln \left( \frac{aAP_q}{rC^p P_x} \right) + \frac{\rho}{\rho^*} (a^* - \rho^* \ln C)}{\frac{B^p}{b} \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p} \\
&+ \frac{\frac{a}{b} \left( \frac{B}{C} \right)^p \ln \left( \frac{B}{C} \right) \rho^*}{\left( \frac{B^p}{b} \right) \left( \frac{aAP_q}{rC^p P_x} \right)^{\frac{p}{p+1}} - \frac{a}{b} \left( \frac{B}{C} \right)^p}
\end{aligned} \tag{A37}$$

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