Time Invariant Decision Rules and Land Development: A Dynamic and Stochastic Analysis*

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Introduction

Natural resource and environmental economists have been interested in the question of (potentially irreversible) land development under uncertainty at least since Weisbrod (1964). Since then, Arrow and Fisher (1974) and Henry (1974) have shed considerable light on this development question. Specifically, these researchers have identified a notion known as option value. The so called Arrow-Fisher-Henry (AFH) notion of option value – sometimes called quasi-option value (QOV) – tells us that when development is both indivisible and irreversible, a landowner who disregards the possibility of procuring new information about the effects of such development will invariably underestimate the benefits of preservation and hence skew the binary choice develop/preserve decision in favour of development.

Does this AFH result hold when the development decision is *divisible*? Epstein (1980), Hanemann (1989), and Batabyal (1999) have studied various aspects of this question and have shown that when the development decision is divisible, the AFH result will not hold in general. One can also inquire about the nature of the development decision when this decision is made in a *multi-period* setting. Because the AFH analysis is conducted in a two period model, the pertinent development question is "Do

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I develop today or tomorrow?" In contrast, in a multi-period setting, the pertinent question is "When do I develop?" This follows from the fact that a landowner's decision problem now is not over two periods but over $n > 2$ periods.

Markusen and Scheffman (1978), Arnott and Lewis (1979), and Capozza and Helsley (1989) have all studied this question in a deterministic environment. However, when the pertinent development decision is irreversible, the use of a certainty framework will bias results about when land ought to be developed. In fact, as we have learned from the investment under uncertainty literature,[1] uncertainty will typically impart an option value to undeveloped land and delay the development of land from, say, agricultural to urban use. Therefore, if we are to comprehend when land ought to be developed in the presence of an irreversibility, it is essential that we explicitly account for uncertainty.

Recently, Titman (1985), Capozza and Helsley (1990), and Batabyal (1996, 1997, 2000) have examined the question of land development under uncertainty. Suppose that the value of vacant land in the first period exceeds the wealth of a landowner who wishes to construct a building. In this setting, Titman (1985) has shown that a wealth maximizing landowner is better off leaving his land vacant. However, the bulk of Titman's (1985) analysis is carried out in a two period model. Consequently, this paper does not really address the multi-period nature of the land development problem. In the context of a "first hitting time" problem,[2] Capozza and Helsley (1990) show that land ought to be converted from rural to urban use at the first instance in which the land rent exceeds the reservation rent. In contrast with this "first hitting time" approach, Batabyal (1996, 1997) has used the theory of Markov decision processes to provide, respectively, a discrete time and a continuous time analysis of the "When do I develop land" question. In both these papers, a specific stopping rule is used to determine when a stochastic "revenue from development" process ought to be halted.

This concludes our brief review of the pertinent extant literature. Now, the objective of this paper is threefold. First, we wish to answer the following question: Given that a landowner observes an intertemporal and stochastic bid (for land development) process, when should he develop his land? In other words, which bid should this landowner accept and thereby terminate the random bid generation process? Second, given the use of a

1. For more on this literature, see Pindyck (1991), Dixit and Pindyck (1994), and Hubbard (1994).
2. For additional details on this, see Dixit and Pindyck (1994: 83-84) and Ross (1996: 363-366).

particular decision rule (see the next section), we wish to determine the average dollar value of the bid that results in land development. Finally, we wish to show how our theoretical results might be operationalized in the specific instance in which the bids are exponentially distributed.

The paper that is most closely related to our paper is Batabyal (2000). In this paper, the "When do I develop land" question is resolved by studying the decision problem of a landowner who wishes to maximize "the *probability* of accepting the best possible offer of development, given that these offers are received sequentially over time" (Batabyal 2000: 150, emphasis in original). However, the reader should note that the theoretical analysis of our paper differs from the Batabyal (2000) paper in particular and the extant literature in general, in three ways. First, we examine the role that *bid* value distributions play in the decision to develop land over time and under uncertainty. Second, we analyze the properties of a hitherto *unstudied* decision rule for a landowner who wishes to obtain a minimum level of revenue from land development.[3] Finally, we determine the *expected value* of the bid that results in land development when the above mentioned decision rule is followed by the landowner.

In the next section, the theoretical framework is formulated and discussed. Then, these theoretical arguments are illustrated with an example based on the exponential distribution function. Finally, some conclusions and suggestions for future research are offered.

The Theoretical Framework

Consider a landowner who owns a plot of land. The decision problem faced by this owner concerns when to develop his plot of land. Keeping with the AFH tradition, we suppose that the development decision is indivisible. In other words, the possibility of partial development of the plot is excluded. The landowner solves his problem in an intertemporal and stochastic setting. The setting is stochastic because the decision to develop depends essentially on the receipt of (dollar valued) *bids* to develop land. These bids arise sequentially over time, in accordance with some independent and identically distributed (iid) stochastic process. In other words, a dollar bid to develop land is received in time period t with a certain probability, independent of other bids. These bids are received over time,

3. This "minimum level of revenue" is sometimes also known as a landowner's "reservation price" (see Barnard and Butcher (1989)). Consequently, in the rest of this paper, we shall use the terms "minimum level of revenue" and "reservation price" interchangeably.

one bid per time period. Consequently, the decision making framework of our paper is dynamic in the sense that this framework requires the landowner to decide when land should be developed on the basis of his observations of the intertemporal and stochastic bid propagation process.

It is important to be clear about the meaning of the last sentence in the previous paragraph. This sentence tells us that our landowner recognizes that he is operating in an *uncertain* environment. Moreover, this landowner also recognizes that his land development decision is based on his observation of a stochastic bid generation process. In this context, the use of the word "observation" means that the landowner understands that the bids are being received over time and that these bids have a *general* distribution function that we denote by *F(•)*. The landowner does *not* know what specific distribution function is represented by *F(•)*. It is only in the subsequent section 3 example that our landowner knows that *F(•)* is the exponential distribution function. The discussion in Barnard and Butcher (1989), Tavernier and Li (1995), and Tavernier *et al* (1996) tells us that there *is* evidence to support our basic contention in this paragraph that landowners contemplating land development operate in an environment of uncertainty. To conclude this discussion, note that the actual source of the bids is not critical to our analysis. It could be the outcome of activities undertaken by private land developers or it could be the result of specific governmental policies.

Upon receipt of a bid, our landowner must decide whether to accept the bid (develop land) or to decline the bid (preserve land) and wait for additional bids. If the landowner accepts a bid, i.e., if he develops his land, then the question of subsequent bids is extraneous. Therefore, the stochastic bid propagation process terminates. We now need to specify a decision rule for our landowner. The simplest decision rule – and one that has not been studied very often in the land development context – is the following: First, suppose that our landowner has a minimum dollar value in mind, say *$A* below which he will not accept a bid, i.e., not agree to have his land developed. In other words, *$A* is the landowner's reservation price. Then, our landowner's decision rule is to accept the first bid that exceeds *$A*.[4]

From the standpoint of our landowner, the situation described in the previous three paragraphs involves acting in a sequential decision making

4. The salience of the reservation price in determining land development has been studied empirically by Barnard and Butcher (1989) and by Tavernier and Li (1995). In addition, there is evidence to suggest that a minimum level of revenue rule or a reservation price rule is used by actual landowners to make development decisions. For instance, Barnard and Butcher (1989) explain land development in Clarke County, Washington, by placing considerable emphasis on the reservation prices of individual landowners.

framework. Suppressing the dollar symbol, let us denote the independent and identically distributed random bids by $B_1, B_2, B_3, B_4 \ldots$. The reader will note that because these bids are dollar valued, they are all positive random variables.[5] Denote the common distribution function of these random bids by $F(\bullet)$. Recall that our landowner will develop his land upon receipt of the first bid whose value exceeds the reservation price A. Formally, the accepted bid is B_N where

$$N = \min \{k \geq 1; B_k > A\} \tag{1}$$

In equation (1), k is an index for the order of the various bids. In other words, B_k, $k = 1$ is the first bid, B_k, $k = 2$ is the second bid and, in general, B_k, $k = 1,2,3,4 \ldots$, is the k^{th} bid received by our landowner.[6] Denote the probability that the first bid B_1 is larger than A by α i.e., $\alpha = Prob\ \{B_1 > A\}$. Our task now is to determine the expected value of the bid that results in the development of our landowner's land. We denote this expectation by $E[B_N]$.

To compute this expectation, first observe that the bid $B_1 = b$ is accepted if $b > A$. Otherwise, if $b \in [0, A]$ then because the bids are independently and identically distributed, our landowner is back where he started. Using this information, we can write the conditional expected value of B_N as

$$E\left[\frac{B_N}{B_1} = b\right] = \begin{Bmatrix} b \text{ if } b > A \\ E\,[B_N] \text{ if } b \in [0, A] \end{Bmatrix} \tag{2}$$

Now, to obtain the unconditional expectation $E[B_N]$, we shall use the law of total probability (see Ross (1996: 21)). This gives

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5. Strictly speaking, instead of using the word "positive," we should be using the word "non-negative." However, in what follows, we shall not worry about this technical detail. After all, in any practical setting, it is highly unlikely that there will be zero valued bids.
6. The reader should note that in the theoretical framework of this paper, the landowner's decision problem is such that it is not meaningful for him to wait – possibly for a very long time - and collect all possible bids and then simply select the highest valued bid. If it were meaningful to do this, then a key objective of this paper (see the Introduction) would become irrelevant. In particular, there would be no point in analyzing a decision rule that involves accepting the *first* bid that exceeds a specific reservation price. Indeed, in this latter situation, one could certainly ask the following question: Why accept the first bid that exceeds the reservation price? Instead, why not accept the bid that exceeds the reservation price by the widest margin?

$$E[B_N] = \int_A^\infty b\, d\, F(b) + (1-\alpha)\, E[B_N] \tag{3}$$

Solving equation (3) for $E[B_N]$ we get

$$E[B_N] = (1/\alpha) \int_A^\infty b\, d\, F(b) \tag{4}$$

Equation (4) gives us the expectation that we are after. This equation tells us that in the general case, the expected value of the bid that results in the development of land equals the product of two terms. The first term is the reciprocal of the probability that the very first bid is accepted by our landowner and hence this first bid results in land development. The second term is the integral of the remaining bids over all possible dollar values that these bids may take.

As indicated in the introduction, studies in the extant literature have analyzed the multi-period nature of the land development question. However, these studies have typically answered the "When do I develop land" question by focusing on the *time* dimension of the underlying question.[7] In contrast, in this paper we have answered the above question by focusing on the *bid* dimension of the question. This focus allowed us to compute the expected value of the bid that results in land development. We now illustrate the formal arguments of this section with an example.

An Example

In this example, we suppose that the bids follow three possible exponential distributions with parameter or rate θ_i, $I = 1, 2, 3$. Further, to demonstrate the impact of alternate values of the rate θ_i on the expected dollar value of the bid that results in land development, without loss of generality, we suppose that $\theta_1 < \theta_2 < \theta_3$ Now, as noted in Taylor and Karlin (1998: 35), the parameter or rate is the reciprocal of the mean of an exponentially distributed random variable. Consequently, from the above ordering of the three rates, it follows that $(1/\theta_1) < (1/\theta_2) < (1/\theta_3)$. In words, the first expo-

7. For a more detailed corroboration of this claim, see Markusen and Scheffman (1978), Capozza and Helsley (1990), and Batabyal (1996, 1997).

nential bid distribution has the highest mean, the second exponential bid distribution has the second highest mean, and the third exponential bid distribution has the lowest mean.

Now, using the properties of the exponential distribution (see Ross (1996, pp. 35-39)), we get $dF(b) = \theta_i \exp(-\theta_i b)\, db$ and $\alpha = \exp(-\theta_i A)$ Let us now use these two pieces of information to simplify equation (4). For each i, $i = 1, 2, 3$ we get

$$E[B_N] = \theta_i \exp(\theta_i A) \int_A^{\infty} b \exp(-\theta_i b) db \quad (5)$$

Now performing the integration in equation (5) and then simplifying the resulting expression, we get a simple equation for $E[B_N]$ Once again, for each i, $i = 1, 2, 3$, that equation is

$$E[B_N] = A + (1/q_i) \quad (6)$$

We are now in a position to interpret equation (6). This equation tells us that when the bids are exponentially distributed, the expected value of the bid that leads to land development equals the sum of the landowner's reservation price A and the mean value of a bid $(1/\theta_i)$ To intuitively see why this result holds, recall that the exponential distribution is unique because it possesses the memoryless property.[8] In our setting this means that conditional on the event that a bid $B > A$, the probability that $B > A$ plus some dollar amount M is the same as the probability that $B > M$.

We can now also see the effect that alternate rate (θ_i) values have on the expected value of the bid that results in land development. In particular, from the discussion in the first paragraph of this section, we see that the *lower* the value of the rate, the *higher* is the mean value of a bid, and hence the *larger* is the expected dollar value of the bid that results in land development. In the framework of this paper, it is not possible for our landowner to choose the rate of the exponential bid distribution. However, if it were possible to do so, our analysis tells us that, *ceteris paribus*, our landowner will always prefer an exponential distribution with a relatively *low* rate. Further, because the constant rate of an exponential distribution is also its constant hazard rate,[9] in terms of the hazard rate, our landowner will – if possible – want to receive bids from an exponential bid distribution with a relatively *low* hazard rate.

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8. For more on the memoryless property of the exponential distribution, see Ross (1996: 35-39) and Taylor and Karlin (1998: 35-36).
9. For on this property, see Taylor and Karlin (1998: 37).

Conclusions

In this paper we modeled the land development problem in an intertemporal and stochastic framework. Using a straightforward decision rule for our landowner, we provided an answer to the "When do I develop land" question. More specifically, we computed the expected dollar value of the bid that results in the development of land and then we illustrated our theoretical reasoning with an example based on the exponential distribution. Our answer to the above question differs from that provided by most of the previous literature on this subject in the sense that we did not focus on the *time* dimension of the multi-period land development question. Instead, we focused on the *bid* dimension of this question.

The analysis of this paper can be extended in a number of directions. In what follows, we suggest two possible extensions. First, the reader will note that we studied a situation in which a landowner recognizes that the stochastic process that he is confronted with is iid in nature. As such, it would be useful to analyze a scenario in which the landowner understands that the random bids that he is faced with are independent but not identically distributed over time. Second, if the landowner learns the statistical properties of the bid propagation process, then it is possible that this landowner will eventually know the specific distribution from which the dollar valued bids are propagated. One could study the consequence of this knowledge – perhaps in a Bayesian framework – on the "When to develop land" question. Studies that analyze these aspects of the problem will provide additional insights into the criteria that govern the development of land over time and under uncertainty.

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