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Migration as a "Catastrophe": Visual Analyses of Four Models

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Abstract.
Catastrophe theory is powerful mathematics that allows both continuous changes in variables and sudden discrete changes in those same variables. It has had a poor reputation as a tool of the social sciences. Economic models of migration assume that agents maximise utility. Utility should change continuously. But migration is a sudden discrete change in location, and causes sudden discrete changes in several key arguments of utility functions. Thus catastrophe theory should nicely describe economic models of migration. Here I show that it does for four economics models of migration. I also show that it has an additional advantage: it allows a seamless integration of secondary migration with the initial, primary move.

Key Words: Migration, mathematical catastrophe, secondary migration.

JEL Codes: J61, C62.

Résumé. La migration vue comme une catastrophe : Des analyses visuelles de quatre modèles
La théorie des catastrophes est un modèle mathématique important qui permet à la fois des changements de variables progressifs et des changements discrets soudains pour ces mêmes variables. Cette théorie n'a pas bonne presse en tant qu'outil pour les sciences sociales. Les modèles économiques de migration supposent que des agents maximisent l'utilité. L'utilité devant changer sans interruption. Mais la migration est un changement de lieu discret et soudain, et elle peut provoquer des changements discrets soudains dans certains arguments clefs des fonctions d'utilité. Par conséquent, la théorie des catastrophes devrait bien décrire les modèles économiques de migration. Dans cet article, je montre que c'est le cas pour quatre modèles économiques de migration. Je montre aussi qu'elle a un avantage supplémentaire: elle permet d'intégrer facilement la migration secondaire au déplacement initial.

Mots clés : Migration, catastrophe mathématique, migration secondaire.

Codes JEL : J61, C62.

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Introduction

Catastrophe Theory

Catastrophe theory was introduced to mathematics with René Thom's 1972 publication of *Structural Stability and Morphogenesis*. Calculus uses variables which change smoothly and continuously. Finite mathematics works with variables that are discrete. "Catastrophe" theory – so named by Thom from the original Greek, meaning an unexpected, dramatic change – is formal mathematics that deals in a unified way with a combination of the two. Catastrophe theory is calculus which involves systems with more than one stable state. Values of variables usually change smoothly but sometimes they make a discrete "jump" from one of those states to another. The jump is the "catastrophe". For his development of these powerful mathematical ideas Thom won the *Grand Prix Scientifique de la Ville de Paris* in 1974.

Almost immediately after its development, catastrophe theory was successfully applied to a large number of physical problems, and it contributed substantial insights to biology: Poston and Stewart (1978). Shortly afterwards there were a few attempts to apply the theory in the social sciences. However, the current view is that applications of catastrophe theory in the social sciences have failed, and the fact that there have been no published applications of catastrophe theory, as such, in the social sciences since the 1970s supports that view.

Poston and Stewart (1978: 409) describe the "problems – and there are many – that are peculiar to the social sciences". The first is that it is often difficult, even impossible, to identify the variables, so variables are often "forced". They also note that, in an attempt to force models into the catastrophe theoretic framework, the models become "unduly complicated [and] simpler models better describe the phenomena" (Poston and Stewart, 1978: 422). A decade later Stewart (1987: 168) summarised: "[Catastrophe theory] has led to a fundamental and extremely important set of mathematical ideas; [but] it resembles the phases of matter: in the physical sciences it is solid, in biology somewhat fluid, in social science possibly gaseous".

A "subtle feature" of catastrophe theory is "the surprising production of big changes in behaviour from small changes in circumstances" (Stewart, 1987: 162). Human migration is a sudden discontinuous change in location and is a "big change in behaviour". In economics, it can be motivated by a small change in circumstances. Thus, economic models of migration seem obvious candidates for a catastrophe theoretic approach.

Here I apply catastrophe theory to four models of migration. My purpose is twofold. First, I rebut Poston and Stewart by showing that catastrophe theory works well in these "social science" models, with none of the problems they describe. The variables are well defined, and catastrophe theory handles the observed patterns neatly, without any forcing. This may be the first fully convincing application of catastrophe theory in social science. Second, this is something of a plea. Applying catastrophe theory to economic models of migration requires that those models employ decision functions which seamlessly integrate primary migration with secondary migration. My plea is that researchers studying migration begin to think about it in the way catastrophe theory requires.

The Fundamental Economic Model of Migration

The individual decision choice in economic migration models since 1962 has always been a variant of the "human capital" model (Sjaastad, 1962). That has individuals moving to increase their lifetime utility, subject to constraints. Basic HK models of migration resolved a number of the anomalies that researchers had found with earlier theory, but it soon began to develop problems. One is the problem of migration in real time. For example, given that an individual moved, if moving is a response to wage/income differentials generating utility differentials, why hadn't he/she moved earlier? Why is migration a steady flow over time, rather than all happening at once? A catastrophe analysis of migration *requires* making the underlying model formally dynamic in real time, and by doing so it not only resolves this problem, it requires of any model that it deal with this problem.

Internal migration – migration *within* a broad geographical unit, usually a nation-state – is characterised by having nearly three-quarters of all *primary* moves being followed by second, *repeat* moves. Some international migration is also characterised by substantial repeat migration. Between one-half and two-thirds of internal repeat moves are moves to a second destination: *onward* moves; the other one-third to one-half are moves back to the location from which the initial move was made: *return* moves[2].

The second problem is that, if the primary move was maximising, why do so many people make multiple moves? A catastrophe analysis of migration – at least a *cusp catastrophe* analysis (see the Section below) – *requires* that the underlying model seamlessly integrate the initial or primary move with the phenomenon of repeat migration. Again, by doing so it not only resolves this problem, it requires of any model that it deal with this problem.

Here, in a catastrophe analysis of four different migration models, I show that catastrophe theory resolves both these problems, and does so in an especially visual way. The first two models are models of *internal* migration, where *internal* means within national borders. The second two are models of *international* migration. The first is general; the last three are explicitly models of *labour market* migration. In the section below I briefly describe the catastrophe; I use the *cusp catastrophe.* In the following section, I use the cusp catastrophe to develop simple visual analyses of four different models of migration. Finally, I summarise and conclude.

[2] These are rough North American figures. Estimates of repeat migration vary, sometimes considerably, across different data sets. DaVanzo and Morrison (1982), using longitudinal data for US states, found that over three-quarters of all *moves* were repeat moves, with about one-third of those being return moves. Vanderkamp (1972), using one-year intervals for Canadian provinces, found that about one-third of all *primary migrants* made a return move in the following year. Newbold (1997), using five-year intervals for both Canadian provinces and US states, found that about half of *primary migrants* made a repeat move; about three-fifths of these were return in Canada, about one-third were return in the US. Newbold's recent work, using smaller time intervals and smaller regions, has found numbers for repeat migration close to those found by DaVanzo and Morrison (1982): see Newbold (1997, 2001) and Newbold and Ciccino (2007).

The Geometry of the Cusp Catastrophe

There are seven catastrophes (Poston and Stewart, 1978): The one that lends itself best to migration is the most commonly applied catastrophe, so common that Poston and Stewart call it "canonical". This is the cusp catastrophe. The cusp catastrophe is the surface containing a complete solution of the maximum and minimum values of the quartic equation:

$$(1)\ V = \mathrm{f}(x; a,b) = x^4/4 + ax^2/2 + bx \qquad \Rightarrow \qquad dV/dx = x^3 + ax + b = 0\ ,$$

where x is the "behavioural" variable, and a and b are the control variables. Poston and Stewart conclude: "Almost the entire description of the equilibrium is geometrically obvious. To complete the picture it is only necessary to distinguish between the stable and unstable equilibria" (Poston and Stewart, 1978: 83). Following convention, I give only the geometry here. The reader needs only to know that the mathematics exists and fully develops all results shown geometrically here.[3]

Figure 1 shows the graph of the cusp itself. The shaded part of the surface are unstable points; the system cannot "be" there. The two control variables are labelled *C1* and *C2*; the behavioural variable is *B*. A convention is to refer to, and treat, *C1* as the "slow" variable and *C2* as the "fast" variable. A catastrophe is most likely to occur from changes in *C2*. Figure 2 is a view of a projection to the *B*, *C2* plane, slicing vertically through the cusp at points **a** and **b**, and **c** and **d**. The distance of that projection along *C2*, like between **a**/**b** and **c/d**, is called "hysteresis".

Figure 3 is the projection of the cusp to the *C1*, *C2* plane. The dotted lines are the projections of the two edges of the fold. The horizontal distance between the dotted lines is the hysteresis. Note that the size of the hysteresis increases as one moves forward on the surface, and decreases as one moves backward on the surface. A catastrophe occurs on the projection when a change in *C2* causes a move across the right-hand dotted line from the left – point **a**/**b** on Figure 3 – or a move across the left-hand dotted line from the right – point **c**/**d** on Figure 3.

On Figure 3, the upper edge of the fold is negatively sloped, and the lower edge of the fold is positively sloped, on the projection to the *C1*, *C2* plane. A "jump" from the upper surface to the lower is caused by a rightward move, i.e. by an increase in *C2*; a "jump" from the lower surface to the upper is caused by a leftward move, i.e. by a decrease in *C2*. An increase in *C1* is a move backward on either the upper or lower surfaces, and, with the shape shown in Figure 3, is an increase in the probability of a jump either down or up. Remember that this is a projection of the figure onto the plane below. So an increase in *C1* is a move backward, and a decrease is a move forward in the three dimensional space.

The shape shown in Figure 3 is not mathematically necessary. The folds can sit in the three dimensional space with both the upper and lower edges of the fold negatively sloped in the projection to the *C1*, *C2* space, or with both the upper and lower edges of the fold positively sloped in the projection. I use the latter in III. C. below; see Figure 5.

[3] Readers interested in the mathematical details can see Poston and Steward (1978) or, of course, Thom (1972). Poston and Stewart (1978) give the mathematical details of the cusp catastrophe on pages 75-89.

Finally, note that in my descriptions I use the words “forward” and “backward”, “right” and “left”, and “up” and “down”. And on the graphs the motions will appear to be forward or backward, right or left, or up or down. But the graphs are just visual depictions of mathematical models, and in the mathematics there is no “forward” or “backward”, “right” or “left”, “up” or “down”. There are only values of variables, and these can only increase or decrease. The words I use are strictly because they make description easier.

FIGURE 1 The Cusp in 3 Dimensions

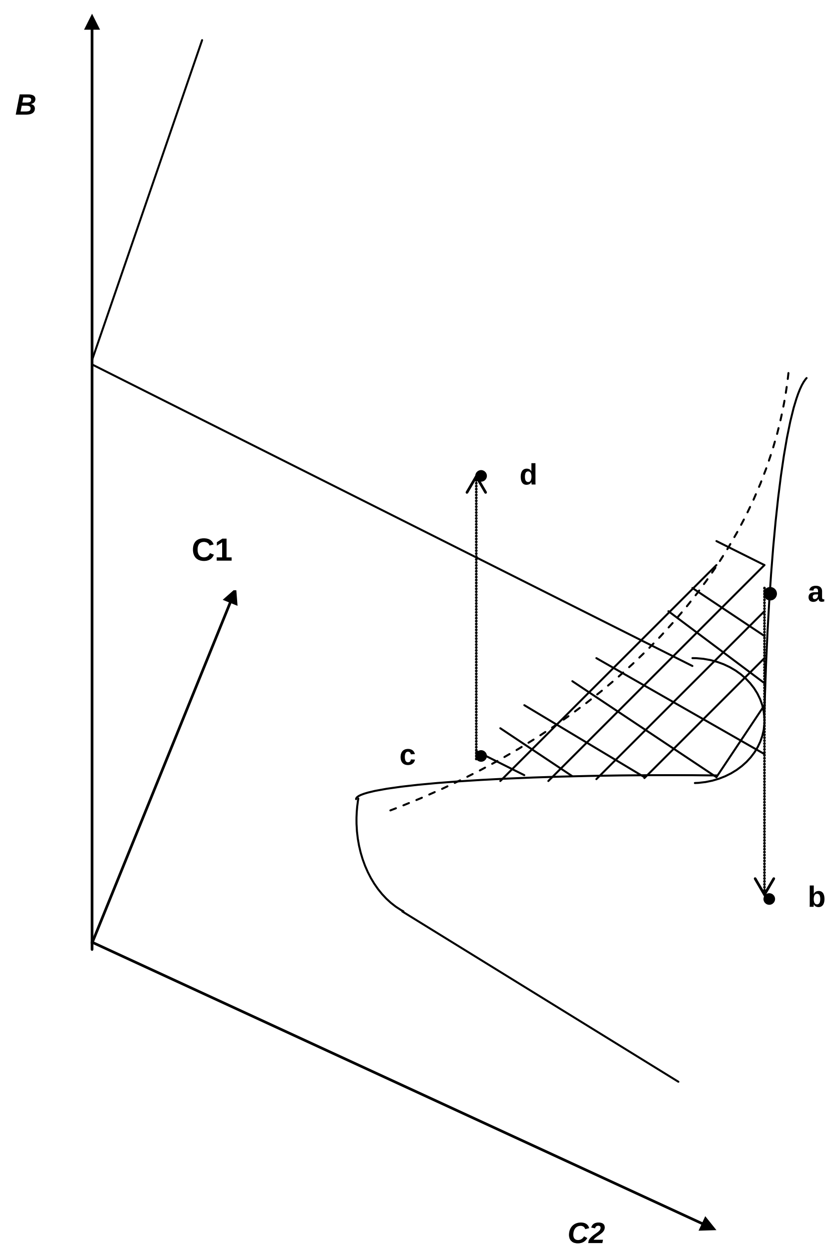


FIGURE 2 The Cusp from the Front

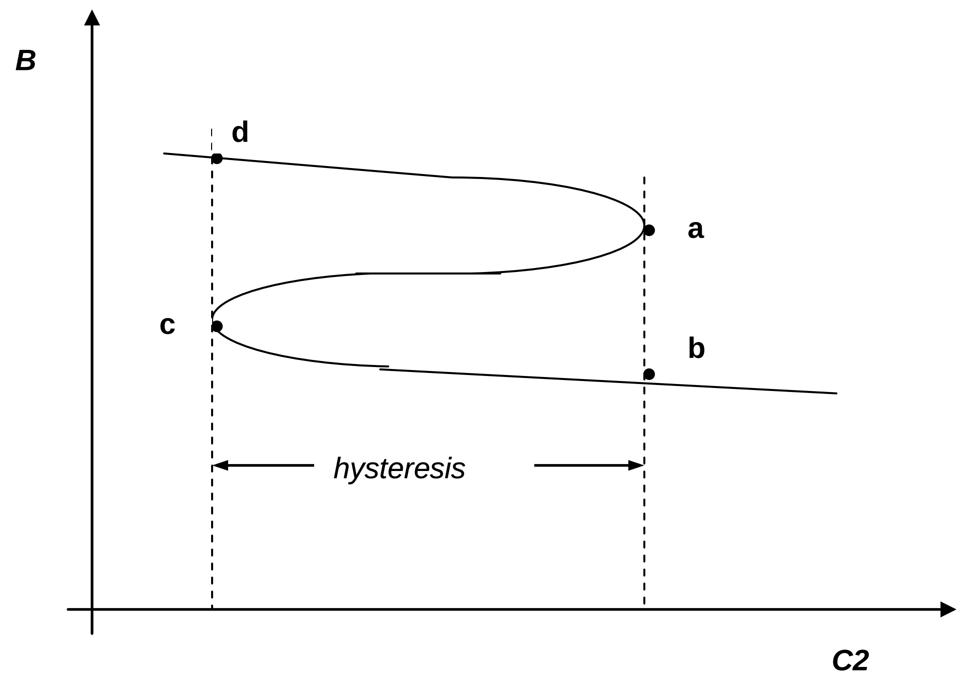
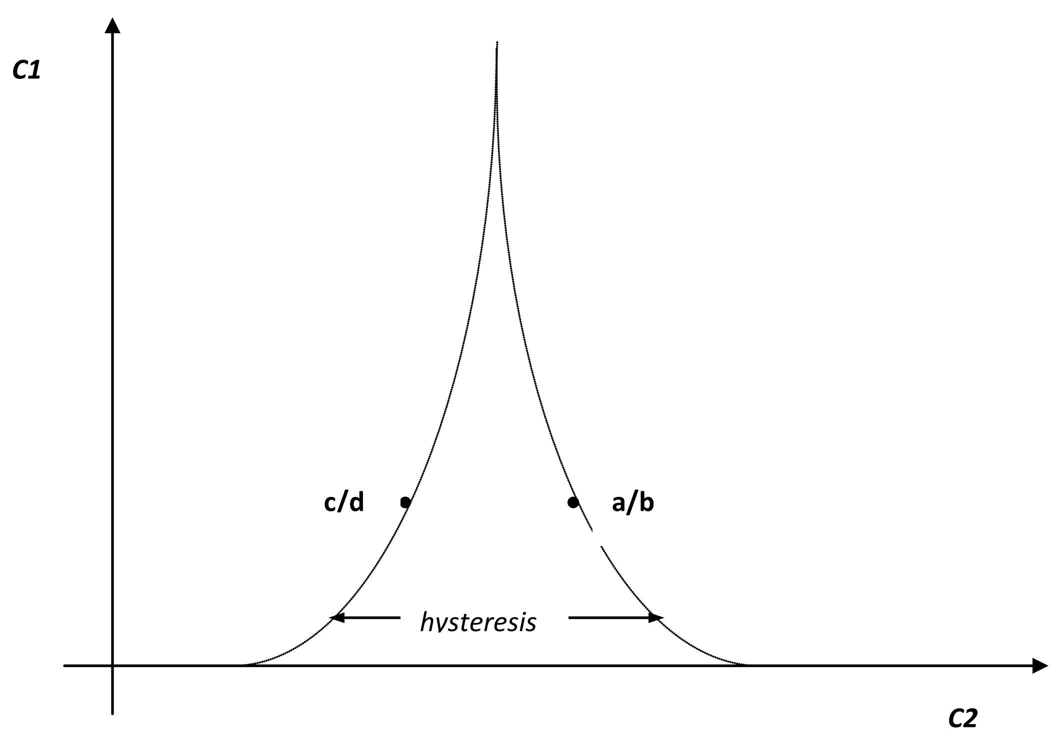


FIGURE 3 The Cusp from Above



Catastrophe Theory and Four Economic Models

Catastrophe Theory and the Fundamental Model

Incomplete Information and disappointment, and location specific capital: First consider the "fundamental" human capital model. In that model, individuals have known costs and returns from moving. If the returns are greater than the costs, they move; otherwise they stay. Treat returns to moving – including "psychic" returns, so this is really lifetime utility – as the fast variable. An increase in returns – a rightward move on the upper surface – could happen for several reasons, all in real time: completion of high school, college, or post-graduate education; or being laid-off or retiring, for example. That rightward move could shift individuals to points like **a** and over to an unstable point, causing a jump down to points like **b**, which is stable. These are primary moves.

But now they become stuck. With costs and returns known ahead of time, there is no clear *internal* reason for returns to decrease quickly in real time, so there is no clear way to model repeat migration. For repeat migration to occur during all states of the labour market, as it does, and for it to be consistent with maximising behaviour, which it must be, something must be happening, and happening quickly and frequently, between the primary move and the repeat move. This problem was resolved by having agents be *incompletely informed*, so that costs and returns are *expected* values; values that not known with certainty and are possibly incorrect.

The idea of incompletely informed agents was initially used by George Stigler in the two papers (1961, 1962) which led to job search theory. Job search theory (and thus implicitly incomplete information) was used by Yezer and Thurston (1976) to develop a formal model of *disappointment* causing return migration. Allen (1979) extended that model by making incomplete information explicit. This is the model of *internal disappointment*, of primary migrants, leading to repeat migration: Once at a destination, primary migrants accumulate more information, and quickly as they search for jobs and/or get acquainted with the destination. If the new information reduces their expected lifetime utility at the destination, they may find that their expected lifetime utility is higher somewhere else. This decision triggers a repeat move. If that somewhere else is "home", it triggers a return move. If it is a third region, it triggers an onward move.[4]

Still missing in 1979 was a full theoretical reason for the size of return migration relative to onward migration. Between one-third and one-half of repeat moves are return moves. At the time of a repeat move there are a very large number of possible destinations for an onward move, but only one for a return move. Why, then, was a return move chosen so frequently when a simple model would suggest the probability of a return move was quite low? The answer to this was given by Julie DaVanzo in 1981: *location-specific capital* (LSK). The idea is simple: LSK "is a generic term denoting any of all of the

[4] The cause of disappointment can also be *external*: some unanticipated radical change in the state of the labour market in locations – for example, a mass layoff at the destination, or a recession in the economy – which causes new migrants to reduce their expected lifetime utility at the destination. The other two parts below use external disappointment.

diverse factors that 'tie' a person to a particular place. It refers to both concrete and intangible assets whose value would be lost or would steadily diminish if the person moved elsewhere" (DaVanzo, 1981: 47).

DaVanzo points out that this implies two propositions: 1) that a return move will have a high probability relative to an onward move because the existence of LSK will make expected lifetime utility higher at "home" than in most other locations, and 2) that the probability of a return move will decline with time after the new move because LSK at "home" will depreciate as it simultaneously accumulates at the destination. As mentioned above, both of these propositions have been supported by virtually all research on repeat migration.

The catastrophe analysis: See Figures 1 and 2. Individuals are situated everywhere on the top surface of Figure 1; their position depends on their individual values of *C1* and *C2*. Individuals have *N* "state dependent" expected lifetime utility functions, with each distinct location being a "state". The states are denoted as *J*, *J* = 1, *N* and the expected utility functions are denoted EU (*J*). Two special locations are differentiated. All individuals are somewhere; call this location "origin" and denote it as *G* (For most individuals "origin" is "home"). All other locations are potential destinations. At any time, one of these destinations has individuals' *maximum* value of expected lifetime utility; denote this destination as *D*. The fast control variable, *C2*, is *K* + [EU(*D*) – EU(*G*)]; *K* is a constant to make the variable always positive. It is "fast" because here it is heavily determined by the flow of information which potential migrants receive steadily in real time[5].

The slow control variable, *C1* is the inverse of moving costs. Some individuals, *potential migrants*, have high values of both *C1* (i.e., low moving costs) and *C2* and are, therefore, situated both farther back on the upper surface and closer to its edge. Quantity of LSK at locations is the behavioural variable, *B*. As a move is made, the quantity of LSK in the migrant's expected lifetime utility function, changes substantially and discretely. This change is normally from greater to lesser for a primary move, and from lesser to greater for a return move.[6]

Changes in [EU(*D*) – EU(*G*)], and therefore the catastrophes, are driven by changes in information. Individuals receive a stream of information in real time which alters their *expectations* of lifetime utility at locations, and moves them right and left on the upper surface. *Positive* information increases EU(*D*) and moves potential migrants to the right on that surface. Some potential migrants will acquire enough positive information to cause [EU(*D*) – EU(*G*)] to be greater than moving costs, moving them to the edge of the upper surface – points like **a** – where they jump to the lower surface – points like **b**. This is a primary move. It discretely reduces the quantity of LSK in the individuals' utility functions; i.e. it jumps individuals to a lower value of *B*. Note that individuals with low moving costs, being situated farther back on the surface, are more likely to be potential migrants.

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[5] All variables are for individuals, so to keep notation simple I do not subscript with "*i*"s; they are implicit.

[6] It would have been possible to use the distance from origin as *B*; that too changes substantially and discretely as a move is made. Instead, the quantity of LSK at locations is used because it is an important argument of the utility functions which control the migration decision.

Once at the destination, new migrants continue to acquire information about the destination, and they acquire it rapidly. If this information is adequately positive or neutral, they stay at the new location. But if more of the information is *negative*, information which *reduces* EU(*D*), new migrants experience *internal disappointment*. There are three responses to internal disappointment: stay and be unhappy, make an onward move, or make a return move. In the latter case, new migrants acquire enough negative information to cause [EU(*D*) – EU(*G*)] to be a larger negative value than moving costs are a positive one, moving them to the edge of the lower surface – points like **c** – where they jump to the upper surface – points like **d**. This is a return move. It discretely increases the value of LSK in the individuals' expected lifetime utility functions; i.e. it jumps individuals to a higher value of *B*.

Note that individuals with low moving costs, being situated farther back on the surface where a small change in expected lifetime utility can trigger a return move, are more likely to make a return move. This conforms to the empirical finding that most return migration – especially quick return migration – is done by young individuals. Note also that the incomplete information model implies that most return migration will be made soon after the primary move. As new information is acquired at a destination, it accumulates on information already acquired. If the new information isn't sufficiently negative to shift the primary migrant quickly across the edge of the lower surface, still more new information added to what has already been accumulated becomes increasingly less likely to do so. In the catastrophe model, this means that movement left and right on the lower surface after the primary move will be fast at first, and then will slow steadily; the prediction from this is that return moves are most likely soon after the primary move. This conforms to the empirical finding.[7]

One can visualize a third fold creating a third surface below the plane containing points like **b** and **c**. Information acquired by primary migrants at destinations – negative about the destination of the primary move, positive about alternative destinations – could then shift them left to a point where they would "jump" to that third surface. This would be an onward move. While this is easy to visualize, I don't show it because it is a little more difficult to draw and the mathematics are considerably more complicated. But the reasoning is identical to that for return moves.

Catastrophe Theory and the "Integrated" Model of Migration over the Business Cycle

The integrated model: The second model to which I apply catastrophe theory is the "integrated" model of migration over the business cycle, where it neatly fits both the pro-business-cyclicality of primary migration *and* the counter-cyclicality of return migration.[8] The pro-cyclicality of primary internal migration is very well documented, with findings going back to the 1920s. Greenwood (1975) surveys earlier findings.

[7] Studies of return migration have found that about 80% of return moves occur within a year of the primary move and almost all occur within three years (Vanderkamp, 1968, 1972; DaVanzo, 1981; DaVanzo and Morrison, 1982).

[8] A detailed description of this model can be found in Allen (2003).

Greenwood, Hunt and McDowell (1986), Jackman and Savouri (1992), Westerlund (1997), and the papers in Padoa-Schioppa (1991) report more recent observations from the US and a large number of European countries. I remind the reader that here only internal migration is being modeled and described.

Curiously, despite recent attention to the time pattern of migration and a burgeoning literature on repeat migration, only five published studies have observed the time pattern of internal return migration, and only one of these is especially recent. Three are of Canada: Vanderkamp (1968, 1972), and Newbold and Liaw (1994). One is of the UK: Bell and Kirwan (1979). One is of migration between Finland and Sweden, which is a single integrated labor market: Kirwan and Harrigan (1986). All but one found return migration to be strongly counter-cyclical, and the exception does not contradict the other findings.[9]

The model here is a formal "integration" of human capital with job search and job matching. As above, information is incomplete, but here incomplete information, as with Stigler, leads to job search. It is included indirectly by having job search formally in the model. Again, individuals have N "state dependent" expected lifetime utility functions, with each location being a "state" indexed as J; $J = 1, N$. The utility functions are denoted U(J).[10] For simplicity, I treat expected lifetime utilities in locations as increasing functions of two variables: Consumption, C, and LSK. Consumption is dependent on earnings (E); while searching (S) it is lower than while working. Denote these as CS and CE; $CS < CE$.

Again, two special locations are differentiated, "origin" (G), and the destination which has each individual's *maximum* utility, (D). The conventional job search approach assumes that the searcher has an *expected* search time in each location; denote this expected search time as $q(J)$. The expected time of leaving the labour force is denoted z. The time periods, t's, are very small, like weeks or days. The expected lifetime utility functions can be written in two parts, one for each of two periods, with expected search time splitting the two, and:

$$\mathrm{U}(G) = \sum_{t=1}^{q(G)} \mathrm{u}[(CS_t(G)+LSK_t(G))_t] + \sum_{t=q(G)}^{z} \mathrm{u}[(CE_t(G)+LSK_t(G))_t] \; ; \qquad (2)$$

$$\mathrm{U}(D) = \sum_{t=1}^{q(D)} \mathrm{u}[(CS_t(D)+LSK_t(D))_t] + \sum_{t=q(D)}^{z} \mathrm{u}[(CE_t(D)+LSK_t(D))_t] \; . \qquad (3)$$

Many individuals will have jobs at "home", will be satisfied with them, and therefore will not be searching. The expected lifetime utility functions of these individuals will be only the second term of equation (2), summed from $t = 1,z$.

The above model is basic human capital, with job search included. Integrating it with job matching requires adding a variable, *hires*, H, which describes the state of the labour

[9] The exception is Newbold and Liaw (1994). But because they use census data, their "contraction" is Canada for the entire period 1981-1986. 1982 was the trough of the "bust", so during most of this period the Canadian economy was actually expanding. The model predicts high levels of return migration right around 1982. The model also predicts *low* levels of return migration from 1983-1986 when employment was rapidly increasing. So Newbold and Liaw's finding of low return migration between 1981 and 1986 does not contradict the model.

[10] All the U's, and the C's and the q's below, are expected values, so for simplicity of notation I leave out the E's. Also, as above, since all variables are for individuals I do have not subscripted with i's.

market.[11] Expected job search time falls as hires rise, and rise and hires fall, so the $q(J)$s are determined by, and are monotonically decreasing in, H.

The catastrophe analysis: Individuals are situated everywhere on the top surface of Figure 1; their position depends on their individual values of *C1* and *C2*. The fast variable, *C2*, is K + [U(D) – U(G)]; K is a constant to make the variable always positive. The slow control variable, *C1*, is again the inverse of moving costs. Some individuals, *potential migrants*, have high values of both *C1* and *C2* and are, therefore, situated both farther back on the upper surface and closer to its edge. And once again, B, the behavioural variable, is quantity of LSK in the utility function.

Changes in [U(D) – U(G)], and therefore the catastrophes, are driven by changes in H. During a business-cycle expansion H rises smoothly in real time, the $q(J)$s fall smoothly, and the expected lifetime earnings from consumption after search increase. This causes the second terms of equations (2) and (3) to increase, and the first terms to decrease. If $CE(D) > CE(G)$, which is true of potential migrants, this in turn causes the value of [U(D) – U(G)] to increase, moving potential migrants rightward on the upper surface. If that value increases enough it will cause [U(D) – U(G)] to be greater than moving costs, moving potential migrants to the edge of the upper surface – points like **a** – where they jump to the lower surface – points like **b** – which is a primary move. Therefore individuals are more likely to make a primary move as H increases. Aggregated over individuals, primary migration rates increase as H increases – the observed pro-cyclicality of primary migration. Note once again that individuals with low moving costs, being situated farther back on the surface, are more likely to be potential migrants.

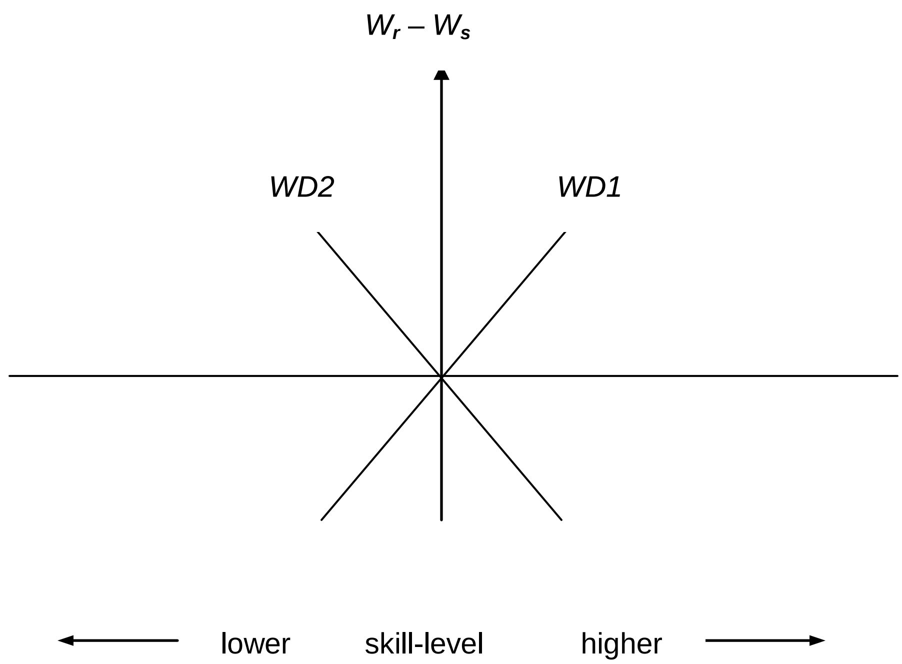
During a contraction the opposite occurs and *external disappointment* takes place. H falls smoothly in real time, the $q(J)$s rise smoothly, and the expected lifetime earnings from consumption after search decrease. This causes the second terms of equations (2) and (3) to decrease, and the first terms to increase. Since $LSK(G) > LSK(D)$, this in turn causes the value of [U(D) – U(G)] to decrease, moving primary migrants leftward on the lower surface. If that value decreases enough it will cause [U(D) – U(G)] to be a larger negative value than moving costs are a positive one, moving primary migrants to the edge of the lower surface – points like **c** – where they jump to the upper surface – points like **d** – which is a return move. Therefore individuals are more likely to make a return move as H decreases. Aggregated over individuals, return migration rates increase as H decreases – the observed counter-cyclicality of return migration. And once again, individuals with low moving costs, being situated farther back on the surface where a small change in [U(D) – U(G)] can trigger a return move, are more likely to make a return move. This again conforms to the empirical finding that most return migration, especially quick return migration, is done by young individuals.

[11] For details on “job matching” and the importance and cyclicality of *hires*, see Blanchard and Diamond (1992), and the papers in Padoa-Schippa (1991).

Catastrophe Theory and the Two Borjas Skill Difference Models: BoW and WoB

Fifteen years ago George Borjas (1994) developed a theoretical approach to international migration where the wage differentials between two national economies are functions of skill level. With no barriers to migration these differentials should be arbitraged to zero, but international migration normally features significant barriers to migration. The rigidities imposed by national immigration quotas and requirements can cause wage differentials to develop and persist. Borjas discusses the two extreme patterns: 1) wage differentials are increasing in skill; and 2) wage differentials are decreasing in skill; these two patterns are shown on Figure 4.

FIGURE 4 Wage Differentials and Skill Levels: Two Patterns



With international migration there is normally a dominant flow. Following Borjas, this means there is a *sending* region and a *receiving* region. Borjas notes that wage differentials increasing in skill-level can be caused by two factors. The first is that the cost of producing skills is lower in the sending region. The second is the existence of immigration quotas that "positively select" workers based on their skill-level. Borjas also notes that wage differentials decreasing in skill-level can be caused by three factors. Two are the reverse of the two above: 1) the cost of producing skills is higher in the sending region, and 2) immigration quotas "negatively select" workers based on their skill-level (For example, *gastarbeiters* in Germany and Sweden). The third is information asymmetries that cause employers in the receiving region to fail to recognise the true skills of individuals from the sending regions.

An interesting feature of the Borjas approach is that return migration is also a function of skill-level. When wage differentials are increasing in skill, the primary migrants most likely to make return moves are "the worst of the best (WoB)"; when wage differentials are decreasing in skill, the primary migrants most likely to make return moves are the "best of the worst (BoW)". These two results flow naturally from the catastrophe model.

The catastrophe analysis of wage differentials increasing in skill-level: Individuals are situated everywhere on the top surface of Figure 1; their position depends on their individual values of *C1* and *C2*. The slow variable, *C1*, is the inverse of skill-level; higher skills are toward the front of the upper surface, and skill-levels decline as one moves backward. The fast variable, *C2*, continues to reflect the business cycle, but here it is simply *hires* in the receiving region: H_r. Changes in H_r drive the catastrophes. The behavioural variable, *B*, is again the quantity of LSK in the (expected) lifetime utility function. (Again, since all variables are for individuals, I do not bother to subscript them with *I*'s).

Two "stylised facts" drive the dynamics of the model: 1) primary migration becomes more attractive when the economy of the receiving region is expanding; 2) (external) disappointment, leading to return migration, is more likely when the economy of the receiving region contracts. These two "stylised facts" are treated as assumptions by Borjas, but they are based on observation, and they are also outcomes of the integrated model developed in Part B above and in more detail in Allen (2003).

For this catastrophe analysis, the upper fold of the cusp must be positively sloped. This is shown on Figure 5, which is the projection of the cusp to the *C1*, *C2* plane. That is, Figure 5 is similar to Figure 3, but with the upper fold angled differently.

With wage differentials increasing in skill-level, potential migrants are the higher skilled because they face greater wage differentials, and they are situated toward the front of the upper surface. As the economy of the receiving region expands, H_r rises. This moves workers to the right on that surface, toward the edge. If they reach the edge – points like **e** on Figure 5 – they jump to the lower surface – points like **f**. This is a primary move, which jumps the individual to a lower value of *B*. The higher the workers' skill-levels, the farther forward they are, the closer they are to the edge, and the more likely they are to make a primary move. Aggregated over individuals, primary migration rates are pro-cyclical *and* positively select higher skills. Workers who are lower skilled are toward the back of the upper surface, so are less likely to be close to the edge, and are therefore less likely to make a primary move.

It is also an outcome of Borjas's model that if workers acquire skills in the sending region, they become more likely to make a primary move. It is an outcome of the analysis here that if workers acquire skills in the sending region, they move forward on the upper surface, moving closer to the upper edge, and thus becoming more likely to make a primary move.

A return move is the jump from **g** to **h** represented on Figure 5. The new migrants who are most at risk to make a return move are those closest to the lower edge of the fold, i.e. those positioned where hysteresis is smallest. Since hysteresis here decreases as skill-level falls, the lower skilled new migrants are closest to the edge of the lower fold. As H_r falls, workers move leftward on the lower surface, and some will reach points like g and jump to points like h on the upper edge; this is a return move. The lower-skilled, being closest to the edge, are most likely to make a return move. Return migration is most likely for the lower skilled (of the higher skilled): the "worst of the best". Return migration selects the WoB.

Catastrophe analysis of wage differentials decreasing in skill-level: Individuals are situated everywhere on the top surface of Figure 1; their position depends on their individual values of *C1* and *C2*. The slow variable, *C1*, is skill-level; lesser skills are toward the front of the upper surface, and skill-levels increase as one moves backward. The fast variable, *C2*, is the same as above: *hires* in the receiving region: H_r . The behavioral variable, *B*, is again the quantity of LSK in the (expected) lifetime utility function[12].

The same two "stylised facts" drive the dynamics of the model as above: 1) primary migration becomes more attractive when the economy of the receiving region is expanding; 2) (external) disappointment, leading to return migration, is more likely when the economy of the receiving region contracts.

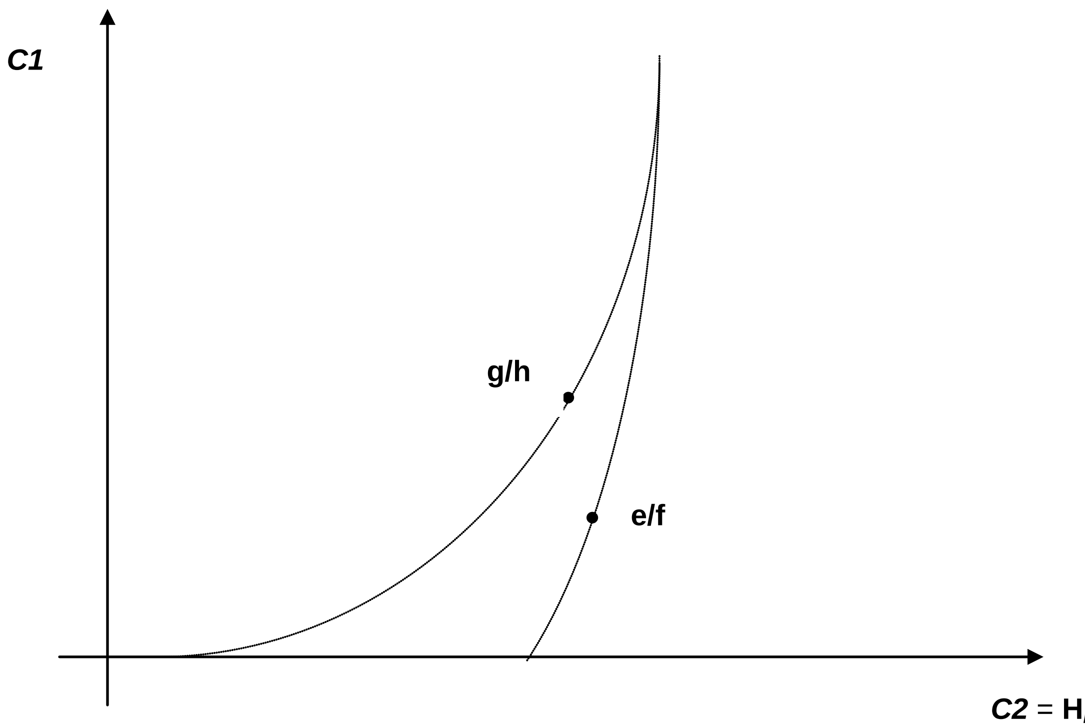
For this catastrophe analysis, the upper fold of the cusp must again be positively sloped. With wage differentials decreasing in skill-level, potential migrants are the lower skilled because they face greater wage differentials, and they are now situated toward the front of the upper surface. As the economy of the receiving region expands, H_r rises. This moves workers to the right on that surface, toward the edge. If they reach the edge – points like **e** on Figure 5 – they jump to the lower surface – points like **f**. This is a primary move, which jumps the individual to a lower value of *B*. The lower the workers' skill-levels, the farther forward they are the closer they are to the edge, and the more likely they are to make a primary move. Aggregated over individuals, primary migration rates are pro-cyclical *and* "positively select" lower skills. Workers who are higher skilled are toward the back of the upper surface, so are less likely to be close to the edge, and are therefore less likely to make a primary move.

It is also an outcome of Borjas's model that if workers acquire skills in the sending region, they become less likely to make a primary move. It is an outcome of the analysis here that if workers acquire skills in the sending region, they move backward on the upper surface, moving farther from the upper edge, and thus becoming less likely to make a primary move.

A return move is the jump from **g** to **h** represented on Figure 5. The new migrants who are most at risk to make a return move are those closest to the lower edge of the fold, i.e. those positioned where hysteresis is smallest. Since hysteresis here decreases as skill-level rises, the higher skilled new migrants are closest to the edge of the lower fold. As H_r falls, workers move leftward on the lower surface, and some will reach points like g and jump to points like h on the upper surface; this is a return move. The higher-skilled, being closest to the edge, are most likely to make a return move. Return migration is most likely for the higher skilled (of the lower skilled): the "best of the worst". Return migration selects the BoW.

[12] Yet again, since all variables are for individuals they are not subscripted with *i*'s.

FIGURE 5 The Cusp from Above: Different Slope



Summary and Conclusion

I have presented a catastrophe analysis of four models of migration. For each model, catastrophe theory offers a simple, visual, depiction. In none of the models are the problems described by Poston and Stewart present. In each the variables are well defined, and catastrophe theory handles the observed patterns neatly, with no forcing. Its application here is not at all "gaseous", it is quite solid.

Poston and Stewart, in discussing one catastrophe model, say "The model does not *require* catastrophe theory …. It can be derived and explained, and will stand or fall, without it. Catastrophe theory merely made it a natural way to think." (Poston and Stewart, 1978: 413). I offer the same conclusion about what I have done here. The four models do not *require* catastrophe theory; they can be derived and explained, and will stand or fall, without it. But the power of applying catastrophe theory is that it makes it "natural" to think in a way that catastrophe theory requires.

Catastrophe theory requires thinking about migration based in real time formal dynamics; in the models here the dynamics of the flow of information in the first, and the changes in hires over the business cycle in the last three. More importantly, catastrophe theory, by providing a visual and seamless integration of the primary move with repeat migration, also requires thinking about primary and secondary migration as different manifestations of a single phenomenon. Primary migration that does *not* result in secondary migration, does *not* do so for reasons that are as important as the reasons for the primary migration decision, and which are as important as the reasons for the second move. It would improve the social science approach to migration if more social scientists viewed migration – literally viewed, with the geometric approach – as a catastrophe. Then the real time dynamics that are necessary in models of migration, and the linkage between the migration and re-migration decisions, which is an observed fact, would be constantly present in their thinking as they developed their models.

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