REGIONAL DEVELOPMENT AND INCREASING RETURNS

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Abstract.
Despite its intuitive appearing, the model of Krugman (1991) is mathematically and empirically intractable; its solutions are, in general, numerical ones obtained using *ad hoc* specifications of the parameters, while its empirical implementation is thwarted by a number of unusual normalisations of the units in which variables are measured. Against this background, this paper develops a one-sector, two-region model in which one region exhibits increasing returns to scale (IRS) which incorporates the concept of distance in the analysis. Our empirical results, which give modest support to the model, encourage further research along the lines of the model.

Résumé. Le développement régional et les rendements (?) croissantes
Depuis Thünen (1824), la géographie économique s'est développé dans le domaine des sciences régionales et un domaine spécialisé en économie nommé la théorie de localisation. Il est curieux que l'aspect spatial du développement économique fut négligée en grande partie par la discipline principale de l'économie (Krugman, 1995). Loin d'être trivial, cette oublie de l'étude de l'espace dans la discipline principale de l'économie trouve ses origines dans les difficultés de la prise en compte du postulat des économies d'échelle dans l'analyse, un défi considérable pour la modélisation théorique avant les années 1970. Il a fallu attendre le progrès innovateur dans la modélisation théorique du milieu des années 1970 avec sa prolifération des études régionales et interrégionales formelles, y compris les études de Dixon et Thirlwall (1975), Faini (1984) et Krugman (1991). Bien que Dixon et Thirlwall (1975) et Faini (1984) aient incorporé les économies d'échelle dans leurs analyses, ils n'ont pas incorporé l'effet de la distance dans leurs modèles. En d'autres mots, leurs régions sont des économies sans espace. Krugman (1991) avait fait la première tentative pour étudier de façon systématique comment une région « centre » industrialisée (avec ses firmes bénéficiant des économies d'échelle) et une « périphérie » agricole (avec ses firmes ayant des rendements constants à l'échelle) peuvent être créée de façon endogène dans un pays. Malgré son intérêt intuitif, le modèle de Krugman (1991) est particulièrement difficile sur le plan mathématique et empirique; ses solutions sont, en général, des solutions numériques obtenues en utilisant des spécifications *ad hoc* de ses paramètres, tandis que sa mise à exécution empirique rencontre des défis de taille à cause du nombre inhabituel de normalisations des mesures avec lesquelles les variables sont mesurées.

   C'est dans ce contexte que cet article élabore un modèle interrégional qui est faisable sur les plans mathématique et empirique et qui utilise le postulat des économies d'échelle dans une région afin d'introduire la distance dans le système en même temps qu'il maintient l'essentiel des études traditionnelles régionales. Nous peaufinerons les

modèles de Guccione et Gillen (1980) et de Gillen et Guccione (1983) qui intègrent les deux modèles de croissance régionale et urbaine, à savoir celui basé sur les exportations et le modèle néoclassique, dans un schéma unifié. Leur postulat de rendements constants à l'échelle dans la production d'un bien homogène, toutefois, exclue essentiellement toute notion réelle de la distance. Notre contribution principale est la généralisation de leur système afin de permettre une région d'être caractérisée par des économies d'échelle. C'est-à-dire, nous postulons que la région à l'étude possède des économies d'échelle tandis que le reste du pays est caractérisé par des rendements constants à l'échelle. De plus, et au contraire de Krugman (1991) qui postule que les économies d'échelle sont dues aux externalités pécuniaires à travers l'effet de liens entre les consommateurs et les industries, nous assumons l'existence des économies d'échelle à cause de l'agglomération du capital dans la région.

Le modèle est calibré en utilisant des données pour la Province de l'Ontario et le Canada de 1947 à 1991, où l'Ontario est définie en tant que le « local » et Canada comme la « nation ». Les résultats empiriques préliminaires fournissent un appui modeste pour notre modèle.

Mots clés : Activité économique régionale, croissance, développement et changement, volume et répartitions spatiales de l'activité économique régionale.

Codes JEL : R11, R12

Introduction

Dating back to Thünen (1824), economic geography has flourished in the study of regional science and a special field in economics called location theory. That the spatial aspect of economic development has been largely neglected by mainstream economics is one of a few puzzles in economic thought (Krugman, 1995).[1] Far from its triviality, the negligence of the study of space in mainstream economics lies in the difficulties in incorporating the assumption of increasing returns to scale (IRS) in the analysis, a formidable task for theoretical modeling before the 1970s. This was so until the path-breaking development in theoretical modeling in mid 1970s, which witnessed the proliferation of formal regional and interregional studies, exemplified by Dixon and Thirlwall (1975), Faini (1984) and Krugman (1991).

Following the tradition of export-based theories, Dixon and Thirlwall (1975) and Faini (1984) examine two different cases of IRS in sustaining inter-regional growth rate differential. Formalizing the verbal model of Kaldor (1970), Dixon and Thirlwall (1975) examined how the Verdoorn effect – a positive relationship between labour productivity and output (or circular causation), contributed to differential in inter-regional growth. In conclusion, Dixon and Thrilwall (1975) argue that, with reasonable parameters in the model, regional growth divergence is unlikely and in all likelihood, constant persistent regional growth rate between regions will prevail. Faini (1984), instead of assuming IRS in the industrial sector as Krugman (1981) did, assumed IRS in the production of non-traded intermediate goods, and examined how this will affect the pattern of trade and sustain cumulative divergence of regional growth rates. One policy implication for regional planners from Faini (1984) is that the underdevelopment of the tertiary sector which produces inputs for industry, not factor

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[1] The great work of Thünen (1824), *The Isolated State*, is the first attempt to derive the general equilibrium response of labour wage and capital rent in cases when factor of production (labour) is fully mobile and commodity flow is costly (Samuelson, 1983).

price distortion, is the main reason for the failure of industrialisation in backward economies. While Dixon and Thirlwall (1975) and Faini (1984) have incorporated increasing returns in their analysis, both of the studies have not considered the effect of distance in their models. In other words, their regions are spaceless economies.

Krugman (1991) contributes the first attempt to systemically study how an industrialised "core" (populated by IRS firms) and an agricultural "periphery" (populated by constant returns to scale (CRS) firms) can be endogenously formed in a country. [2] Essentially Krugman (1991) uses the assumptions of monopolistic competition, non zero transportation cost and immobile agricultural workers to make IRS consistent with a non-extreme equilibrium. Krugman (1993) generalises the framework of Krugman (1991) to illustrate that concentration of population could appear in a variety of locations, i.e., there are multiple equilibria for metropolitan locations. Despite its intuitive appeal, the model of Krugman (1991) is mathematically and empirically intractable; its solutions are, in general, numerical ones obtained using *ad hoc* specifications of the parameters, while its empirical implementation is thwarted by a number of unusual normalisations of the units in which variables are measured. Fujita and Krugman (2004) describe that the core elements of Krugman (1991) as "Dixit-Stiglitz, icebergs, evolution and the computer". In other words, numerical stimulations by computer are used to generate the outcomes predicted by the models.

Against this background, this paper develops a mathematically and empirically tractable interregional model that uses the assumption of IRS in one region to introduce distance into the system while still retaining the basic flavour of traditional regional study. We extend the model of Guccione and Gillen (1980) and Gillen and Guccione (1983) which integrate the two models of regional and urban growth, namely the export-base and the neoclassical type system, into a unified framework. Their assumption of constant returns to scale (CRS) in the production of one homogenous good, however, essentially excludes any meaningful notion of distance. Our contribution thus lies in generalising their system to allow one region to exhibit IRS. That is, we assume that the region under study has IRS while the rest of the country operates under CRS. Also, unlike Krugman (1991) who assumes IRS due to pecuniary externalities through linkage effects among consumers and industries, we assume IRS due to agglomeration of capital in the region.

The remainder of the paper is organised as follows. The next section presents our model which extends the model of Guccione and Gillen (1980) to allow for IRS in one region. We discuss the existence of a non-extreme equilibrium and analyse the short run and long run equilibria. Then, we empirically test our model using data from Canada. Finally, we present our conclusions.

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[2] In the past, all the studies assumed that there is a centre and explained why other periphery regions will develop. The significance of Krugman (1991) in initiating a new field called 'new economic geography' can be appreciated by the fact that both *Spatial Economics II* (2005), edited by Masahisa Fujita, and *New Economic Geography* (2005), edited by J. Vernon Henderson, put Krugman (1991) as the first paper in their volumes.

The Model

Consider a country which could be divided into two regions: one with increasing returns to scale (IRS) – hereinafter "the local" – and the other with constant returns to scale (CRS) – hereinafter "the nation". These two regions produce one homogenous product by means of two inputs (capital K and labour E). The analysis is divided into two stages – the short-run and the long-run, according to the adjustment of the capital stock. Let the local and national output be X and $\overline{X}$ respectively, and let units of labour and capital utilised be $E, \overline{E}$, K and $\overline{K}$ respectively. Price is denoted by P for the local and $\overline{P}$ for the nation. The production functions are assumed to be Cobb Douglas, the local one being

$$X = \alpha_1 E^{\alpha_2} K^{\alpha_3} \tag{1}$$

and the national one being

$$\overline{X} = \alpha_1 \overline{E}^{\alpha_2} \overline{K}^{1-\alpha_2} \tag{2}$$

where $\alpha_1, \alpha_2, \alpha_3 > 0, 1 - \alpha_2 < \alpha_3 < 1$ and $\alpha_2 < 1$.

To retain competitive behaviour by local firms, we assume that IRS is external to the firm. If IRS is internal to local firms, eventually there will be only one local producer. Also, Krugman (1998) points out that both plants and firms in large cities tend to be smaller than those in small cities, which provide supportive evidence that cities may not be sustained by increasing returns at the plant level.

There are a large number of firms in the 'local', each of which has the same production function as firm in the nation augmented by local capital stock. That is

$$X_i = \alpha_1 E_i^{\alpha_2} K_i^{1-\alpha_2} K^u$$

where i denotes the ith firm and $u>0$. That is, the productivity of each firm is enhanced beyond the level of firm in the nation by the agglomeration of capital in the local economy. Competitive pressure ensures that each local firm is a price-taker and, in maximising profit, neglects its effect on total local capital stock. The cost function of the local firm is

$$C_i = \frac{r^{1-\alpha_2} w^{\alpha_2} X_i}{\alpha_1 \alpha_2^{\alpha_2} (1-\alpha_2)^{1-\alpha_2} K^u}$$

which implies the following perfectly elastic supply

$$P = \frac{r^{1-\alpha_2} w^{\alpha_2}}{\alpha_1 \alpha_2^{\alpha_2} (1-\alpha_2)^{1-\alpha_2} K^u} \tag{3.}$$

Thus, the local supply increases (the supply price decreases) with capital stock, i.e., the aggregate local economy exhibits IRS but the individual firms act as perfect

competitors. Since all local firms have identical production function and factor prices, we have $\dfrac{E_i}{K_i} = \dfrac{E}{K}$ for all i. Hence, observationally, the aggregate local production function is

$$X = \sum X_i = \sum \alpha_1 (\frac{E_i}{K_i})^{\alpha_2} K_i K^u = \alpha_1 (\frac{E}{K})^{\alpha_2} \sum K_i K^u = \alpha_1 E^{\alpha_2} K^{1-\alpha_2 + u}$$

where the summation is over all local firms. Defining $\alpha_3 = 1 - \alpha_2 + u$ yields the local production function (1).

 With no further restrictions, the above model will lead to an extreme and stable equilibrium; increasing returns to scales in the local economy would eventually yield a price P lower than $\overline{P}$ and satisfy the whole national demand, a case similar to the extreme points and stable equilibrium of Fain (1984) and Krugman (1991). To provide more interesting analysis, we assume that the time to attain this extreme solution is much longer than the "long run" in the model. Moreover, we introduce a non zero iceberg cost of transportation, δ per dollar of product per unit of distance, paid by the consumer so that the condition $P < \overline{P}$ for local monopolisation becomes $P(1 + \delta d) < \overline{P}$ for all distances d.[3] The existence of local IRS ensures that, for a sufficiently large amount of local capital stock, this last condition will be met eventually. It is assumed, however, that local growth is not instantaneous but is constrained in finite time. Thus, at any point in time (and hence for all empirical observations) there exists a distance d_t such that $P(1 + \delta d_t) > \overline{P}$ and the nation has a positive (but declining) share of total production. Analytically, this "equilibrium" is similar to the "moving equilibria" of Young (1928).[4] Under the assumption of the existence of temporal non-extreme equilibrium, we revise the Guccione and Gillen (1980) C-model and consider the growth path of the local region. For this purpose, it is assumed that the real wage rates are equal in the two regions in both the short and long run, i.e. $\dfrac{w}{p} = \dfrac{\overline{w}}{\overline{P}}$ where w, $\overline{w}$ denote the local and national wage rates respectively.

Short-run analysis when capital stocks are fixed

Firms maximise their profit with respect to employment. The FOC for the ith local firm is

$$P\alpha_1 \alpha_2 E_i^{\alpha_2 - 1} K_i^{1-\alpha_2} K^u = w$$

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[3] The concept of iceberg cost was introduced into spatial economic analysis by Samuelson (1952). The right-hand side (RHS) of this inequality does not include transport costs since the nation produces under CRS.

[4] He also thought "the apparatus which economics have built up for the analysis of supply and demand in their relations to prices does not seem to be particularly helpful for the purposes of an inquiry into these broader aspects of increasing returns" (Young, 1928: 533).

which in the aggregate local economy yields

$$P\alpha_1\alpha_2 E^{\alpha_2-1} K^{\alpha_3} = w \qquad (4)^5.$$

Similarly for the nation, we have

$$\overline{P}\alpha_1\alpha_2 \overline{E}^{\alpha_2-1}\overline{K}^{1-\alpha_2} = \overline{w} \qquad (5).$$

Conditions (4) and (5) imply

$$E = \frac{K^{\frac{\alpha_3}{1-\alpha_2}}}{\overline{K}}\overline{E} = \theta\overline{E} \qquad (6).$$

Since K and $\overline{K}$ are fixed, their ratio θ will be a constant. This relation gives the employment function of the export-base model.

Long-run analysis where capital can be adjusted

For the nation, the First Order Condition (FOC) for profit maximisation subject to equation (2) yield the factor proportion

$$\frac{\overline{K}}{\overline{E}} = \frac{1-\alpha_2}{\alpha_2}\frac{\overline{w}}{\overline{r}} \qquad (7)$$

and the factor price frontier

$$\overline{P}\alpha_1\alpha_2(\frac{1-\alpha_2}{\alpha_2})^{1-\alpha_2}(\frac{\overline{r}}{w})^{\alpha_2-1} = \overline{w} \qquad (8)$$

where $\overline{r}$ denotes the rental value of capital, which is assumed to be equal across regions. The FOC for cost minimisation by local firm i, subject to its production function, can be written as $\frac{K_i}{E_i} = \frac{(1-\alpha_2)}{\alpha_2}(\frac{w}{\overline{r}})$ since the firm ignores its effect on the term K^u. Aggregating this last relation over all i yield a local factor proportion equation of the same form as the nation's, specifically,

$$\frac{K}{E} = \frac{1-\alpha_2}{\alpha_2}\frac{w}{\overline{r}} \qquad (9).$$

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[5] It is easy to check that the second order condition is satisfied.

The factor price frontier for the local, given by equation (3), differs from that for the nation in that it contains the term K^{-u}. This causes the local optimal price to fall as capital accumulates. Hence the equilibrium distance satisfies $P(1 + \delta d) = \overline{P}$, that is,

$$d = \frac{\overline{P} - P}{\delta P} \tag{10}$$

varies. We turn now to model this variation.

Let the market be located on a straight line between 0 and 1, with the local at point 0. Owners of capital are assumed to locate evenly along [0,1] while workers locate at the point of their employment. Since it is assumed that consumers pay the transport cost, workers will spend their wages at the point of their employment. Thus the value of the worker demand for the local output is wE while that for the nation's output is $\overline{wE}$. The total expenditure of capital owners $(\overline{rK} + \overline{rK})$ will be distributed between the local and the nation in the ratio $\dfrac{d}{1-d}$. It follows that the value of demands for local and national output are $PX = wE + d\overline{r}(K + \overline{K})$ and $\overline{PX} = \overline{wE} + (1 - d)\overline{r}(K + \overline{K})$ respectively. These relations require that

$$\frac{PX - wE}{\overline{PX} - \overline{wE}} = \frac{d}{1 - d}$$

The FOC conditions (3), (7), (8) and (9) imply that the left-hand side (LHS) of the latter equality is $wE/\overline{wE}$; hence, using condition (10), we have

$$\frac{wE}{\overline{wE}} = \frac{\overline{P} - P}{\delta P - (\overline{P} - P)} \tag{11}.$$

Equation (11) essentially allows the equilibrium at a point in time to shift as $\dfrac{\overline{P}}{P}$ varies. The model that defines this moving equilibrium is completed by the requirement that the real wage rate be equal at all points of production, the assumption of full employment and exponential total population growth, and by an appropriate specification of the exogenous variables. Thus we assume

$$\frac{w}{P} = \frac{\overline{w}}{\overline{P}} \tag{12},$$

$$E = \gamma N \tag{13}, \text{ and}$$

$$\overline{E} = \gamma \overline{N} \tag{14},$$

where γ is the labour force participation rate and N and $\overline{N}$ are local and national population respectively. Finally, population is assumed to grow exponentially, i.e.,

$$N + \overline{N} = n_0 e^{n_1 t} \tag{15}.$$

The six-equation system given by Equations 1, 3, 9, 11, 12, and 13 is a complete regional model. That is, its solution yields the local variable X, P, w, E, K and N in terms of the national ones. The national sector (Equation 2, 7, 8, and 14) does not determine all national variables; instead, two of them must be exogenous. It is assumed that $\overline{P}$ and $\dfrac{\overline{r}}{\overline{P}}$ are exogenous. The former is determined by monetary policy (the model determines only relative national prices), while the later is determined internationally (i.e., the supply of capital to the local and the nation is perfectly elastic at the national real capital rental value). For the purpose of describing equilibrium at a point in time, $\overline{N}$ is also exogenous, but relation (15) provides the equation of motion for the interregional system, i.e. population growth shifts the equilibrium (recall that the supply of the other input, capital is perfectly elastic) over time.

The national equilibrium at a point in time is easily shown to be:

$$\overline{E} = \gamma \overline{N} \tag{16}$$

$$\frac{\overline{w}}{\overline{P}} = [\alpha_1 \alpha_2^{\alpha_2} (1-\alpha_2)^{1-\alpha_2}]^{\frac{1}{\alpha_2}} (\frac{\overline{r}}{\overline{P}})^{-(\frac{1-\alpha_2}{\alpha_2})} \tag{17};$$

$$\overline{K} = \gamma [\alpha_1 (1-\alpha_2)]^{\frac{1}{\alpha_2}} (\frac{\overline{r}}{\overline{P}})^{\frac{-1}{\alpha_2}} \overline{N} \tag{18};$$

$$\overline{X} = \gamma \alpha_1^{\frac{1}{\alpha_2}} (1-\alpha_2)^{\frac{1-\alpha_2}{\alpha_2}} (\frac{\overline{r}}{\overline{P}})^{-\frac{1-\alpha_2}{\alpha_2}} \overline{N} \tag{19}.$$

The regional model of the local economy can be obtained as follows. Equations (3), (8) and (12) yield

$$\frac{P}{\overline{P}} = K^{-\frac{u}{1-\alpha_2}} \tag{20}$$

while relations (7), (9) and (11) imply that

$$K = (\frac{\overline{P}/P - 1}{1 + \delta - \overline{P}/P}) \overline{K} \tag{21}.$$

These last two equations define (implicitly) P and K as function of the national variables $\overline{P}$ and $\overline{K}$. Equation (12) yields

$$w = \overline{w}/(\frac{\overline{P}}{P})$$

(22),

and, using relation (9) gives

$$E = \frac{\alpha_2}{1-\alpha_2}(\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})\frac{\overline{P}}{P}\frac{\overline{r}}{\overline{w}}\overline{K}$$

(23).

Finally, the regional model requires that

$$N = \frac{1}{\gamma}\frac{\alpha_2}{1-\alpha_2}(\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})\frac{\overline{P}}{P}\frac{\overline{r}}{\overline{w}}\overline{K}$$

(24),

and

$$X = \alpha_1(\frac{\alpha_2}{1-\alpha_2})^{\alpha_2}(\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})^{\alpha_2+\alpha_3}\overline{K}^{\alpha_2+\alpha_3}(\frac{\overline{P}}{P}\frac{\overline{r}}{\overline{w}})^{\alpha_2}$$

(25).

Equilibrium (at a point in time) for the interregional system is obtained by substituting the national one into the regional. This yields the following functions of the exogenous variables $\overline{P}$, $\overline{r}/\overline{P}$ and $\overline{N}$, where $f(\overline{P},\overline{K})$ denotes the solution for P obtained from (20) and (21);

$$P = f(\overline{P},\gamma[\alpha_1(1-\alpha_2)]^{\frac{1}{\alpha_2}}(\overline{r}/\overline{P})^{-\frac{1}{\alpha_2}}\overline{N})$$

(26);

$$K = (\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})\gamma[\alpha_1(1-\alpha_2)]^{\frac{1}{\alpha_2}}(\overline{r}/\overline{P})^{-\frac{1}{\alpha_2}}\overline{N}$$

(27)

$$w = P[\alpha_1\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}]^{\frac{1}{\alpha_2}}(\overline{r}/\overline{P})^{-\frac{1-\alpha_2}{\alpha_2}}$$

(28);

$$E = \gamma(\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})\frac{\overline{P}}{P}\overline{N}$$

(29);

$$N = (\frac{\overline{P}/P-1}{1+\delta-\overline{P}/P})\frac{\overline{P}}{P}\overline{N}$$

(30); and,

$$X = \gamma^{\alpha_2+\alpha_3} \alpha_1^{\frac{\alpha_2+\alpha_3}{\alpha_2}} \alpha_2^{\alpha_2} (1-\alpha_2)^{-\alpha_2+\frac{\alpha_3}{\alpha_2}} (\frac{\overline{P}/P-1}{1+\delta-\overline{\overline{P}}/P})^{\alpha_2+\alpha_3} (\overline{r}/\overline{P})^{\alpha_2-\frac{\alpha_3}{\alpha_2}} \frac{\overline{P}}{P} \overline{N}^{\alpha_3} \qquad (31).$$

Equations 16 to 19 and 26 to 31 describe the interregional equilibrium at a point in time. Finally, the moving equilibrium can be derived by using assumption (15) and relation (30) to obtain

$$\overline{N} = n_0 (1+ \frac{\overline{P}/P+1}{1+\delta-\overline{\overline{P}}/P} \frac{\overline{P}}{P})^{-1} e^{n_1 t} \qquad (32)$$

and substituting this results in temporal interregional equilibrium.

Data Analysis

In the empirical section, we will estimate two equations which constitute the core of the model, i.e., equations specifying that the local population and employment relative to that in the nation (including the local), which increase as the cost of capital decreases and increase (slowly) over time. Using a log-linear Taylor approximation of the two functions, we will estimate the following regression. See Appendix 1 for the details of Taylor approximation. The reduced form equations that are of particular interest here are:

$$Ln(\frac{N}{N+\overline{N}}) = \beta_0 + \beta_1 t + \beta_2 Ln(\frac{\overline{r}}{\overline{P}}) = Ln(\frac{E}{E+\overline{E}})$$

where

$$\beta_0 = \frac{1-\alpha_2+u}{1-\alpha_2+u-(1-\lambda_1)(1+c_1)u}[Ln(n_0/\lambda_0)+(1-\lambda_1)Lnc_0]$$

$$- Lnn_0 + \frac{(1-\lambda_1)(1+c_1)u}{1-\alpha_2+u-(1-\lambda_1)(1+c_1)u}Ln[\gamma\alpha_1^{\frac{1}{\alpha_2}}(1-\alpha_2)^{\frac{1}{\alpha_2}}],$$

$$\beta_1 = \frac{(1-\lambda_1)(1+c_1)u}{1-\alpha_2+u-(1-\lambda_1)(1+c_1)u}n_1,$$

$$\beta_2 = \frac{(\lambda_1-1)(1+c_1)u}{[1-\alpha_2+u-(1-\lambda_1)(1+c_1)u]\alpha_2}.$$

Notice first that if $u=0$, i.e., the region has CRS, then these equations collapse to the Borts-Stein-Muth neoclassical model in which $\frac{N}{N+\overline{N}}$ which and $\frac{E}{E+\overline{E}}$ are constant. Then, with approximate specifications of adjustment mechanisms the Guccione and Gillen models are special cases of the one formulated here.

Except for implausibly large values of α_3 relative to $1-\alpha_2$, the quantity $1-\alpha_2+u-(1-\lambda_1)(1+c_1)u$ will be positive. Hence we restrict $1-\alpha_2+u-(1-\lambda_1)(1+c_1)u$ to be positive. With this constraint we have the restrictions $\beta_1 > 0$, $\beta_2 < 0$. That is, the region's share of total population and employment increases over time and increases as the rental value of capital decreases. The former implication captures the movement of the static equilibrium as the population increases. The latter restriction arises from the increase in the static equilibrium's capital stock (when the cost of capital declines) combined with the assumption that the region's IRS is caused by capital agglomeration.

Finally, from a statistical perspective, the model is a touch "suicidal". The restrictions are $\beta_1 > 0$ and $\beta_2 < 0$ but if $|\beta_1|$ or $|\beta_2|$ are very large then the region will quickly dominate the nation – a result that is inconsistent with causal observation and with (the resulting) assumption that the extreme equilibrium solution is not observable in the existing data. Hence statistically we are faced with the problem of distinguishing between small variations of β_1 and β_2 from zero (i.e., the IRS case) and zero values of both parameters (i.e., the Bort-Stein-Muth case). Given this fact together with the well known difficulties of measuring the rental value of capital, and the compactness of the model, the preliminary empirical results are encouraging (although not sterling).

We estimate the model using data for the Ontario Province and Canada from 1947 to 1991, where Ontario is defined to be "the local" and Canada to be "the nation". Canada has eleven provinces and three territories. The difference between a province and a territory is the source through which they receive the power. Whilst a province receives the power from the Crown, a territory receives its power and authority from the federal government. The process of federation began in 1867 when Ontario, Quebec, New Brunswick, and Nova Scotia federated into the Dominion of Canada. Other provinces, including British Columbia and Manitoba, continued to join in. In 1949, Newfoundland and Labrador became a province of Canada. Table 1 presents some information of the provinces and territories of Canada. We choose Ontario to be the local because Ontario is the largest province by population and second largest, after Quebec, in total area in Canada. Interestingly, historical incidents determine the development path of Ontario. For example, until the early 1970s, Quebec was the most prosperous province in Canada. However, political instability due to the movement of independence led to massive outflow of capital from Quebec to Ontario, making Ontario the financial and technology centre in Canada. Two important cities in Canada, namely Ottawa, the capital of Canada, and Toronto, the largest city in Canada, are located in Ontario.

We obtain the employment data from Statistics Canada, Annual Review of Employment and Payrolls. Population data were obtained from Statistics Canada, Canadian Economic Observer. The exogenous variable $\frac{\bar{r}}{P}$ is measured by a 12 month average of the industrial composite common stock yields obtained from the Historical Statistics of Canada and the Bank of Canada Review. Table 2 presents the regression results.

TABLE 1 Provinces of Canada

Province/territories	Capital	Entered Confederation	Population (2008)	Area (km^2)
Ontario	Toronto		12,891,787	1,076,395
Quebec	Quebec City		7,744,530	1,542,056
Nova Scotia	Halifax	July 1, 1867	935,962	55,284
New Brunswick	Fredericton		751,527	72,908
Manitoba	Winnipeg	July 15, 1870	1,196,291	647,797
British Columbia	Victoria	July 20, 1871	4,428,356	944,735
Prince Edward Island	Charlottetown	July 1, 1873	139,407	5,660
Saskatchewan	Regina	1-Sep-2005	1,010,146	651,036
Alberta	Edmonton		3,512,368	661,848
Newfoundland and Labrador	St. John's	31-Mar-1949	508,270	405,212
Northwest Territories	Yellowknife	July 15, 1870	42,514	1,346,106
Nunavut	Iqaluit	1-Apr-1999	31,152	2,093,190
Yukon	Whitehorse	June 13, 1898	31,530	482,443

TABLE 2 Regression Results

Dependent variable	$\ln(N/N+\overline{N})$			$\ln(E/E+\overline{E})$	
T	0.00235	0.00029		0.0018	0.00043
	(17.08)***	(1.767)*		(13.33)***	(1.987)*
$\ln(\overline{r}/\overline{P})$	-0.0166	-0.0106		-0.0245	-0.0046
	(2.065)**	(2.986)**		(3.075)**	(.743)
$\ln(N_{-1}/N_{-1}+\overline{N}_{-1})$		0.859			0.737
		(13.454)***			(6.977)***
Constant	-1.009	-0.094		-0.869	-0.237
	(20.377)***	(1.263)		(17.788)***	(2.455)**
R^2	0.906	0.983		0.868	0.94
DW statistics	0.282	1.654		0.725	1.878

Notes: Absolute value of t statistics in parentheses* significant at 10%; ** significant at 5%; *** significant at 1%.

The positive coefficients of time *t* and the negative coefficients of $\ln(\dfrac{\overline{r}}{\overline{P}})$ fit the restrictions well. Comparing these two estimated equations, the estimated value of β_0 of the population equation is 1.16 times of that of the employment equation, and the

estimated value of β_1 and β_2 are 1.298 times and 0.68 time separately. Clearly it may require finding some way to measure the degree of these difference and similarity. All the coefficients appear to be significant at the 10% level. The coefficients of determination are reasonable, and the appropriate F statistics are large enough to indicate the model is structurally stable. However the low Durbin-Watson statistics suggest the presence of serial correlation. Thus, we follow Gillen and Guccione (1983) and introduce an adjustment mechanism of the form

$$Ln(\frac{N_t}{N_t + \overline{N}_t}) - Ln \frac{N_{t-1}}{N_{t-1} + \overline{N}_{t-1}}) = \eta[Ln(\frac{N_t^*}{N_t^* + \overline{N}_t^*}) - Ln \frac{N_{t-1}}{N_{t-1} + \overline{N}_{t-1}})]$$

where superscript * denotes the model's prediction and where $0 < \eta < 1$. The resulting equation has the form

$$Ln(\frac{N_t}{N_t + \overline{N}_t}) = \eta_0 + \eta_1 t + \eta_2 Ln(\frac{\overline{r}}{P}) + \eta_3 Ln(\frac{N_{-1}}{N_{-1} + \overline{N}_{-1}})$$

where the subscript -1 indicates a one-period lag, and where $\eta_1 > 0$, $\eta_2 < 0$ and $0 < \eta_3 < 1$. The equation for $Ln(\frac{E}{E + \overline{E}})$ has the same form but, since the speeds of adjustment of employment and population may vary, the magnitudes of the coefficients may differ. The second and fourth columns of Table 2 present the regression results with this adjustment term respectively for relative population and employment. The coefficients on the term $Ln(\frac{\overline{r}}{P})$ of the employment equation and the constant of the population equation are insignificant at the 10% level, but all remaining coefficients are significant at the 10% level. The signs of the coefficients also satisfy the restrictions. Since the Durbin Watson test is not valid when there is a lagged dependent variable in the equations, Durbin's (1970) h-statistics were tried. The test is carried out by referring $h = (1-DW/2)(T/1-TS^2))^{1/2}$, where S^2 is the estimated variance of the least squares regression coefficient on the one-period lagged dependent variable and T is the number of observations, to standard normal tables. Large values of h lead to rejection of the non-autocorrelation hypothesis. The h statistics for the population and employment equations are 1.286 and 0.579 respectively, which indicate that there is no autocorrelation problem. Thus, after the introduction of an adjustment mechanism, the preliminary empirical results lend modest support for our model.

Conclusion

In this paper we developed a one-sector, two-region model in which one region exhibits IRS and incorporated the concept of distance into the analysis. The development of the local IRS economy is shown to be constrained to a "moving equilibria" path. Our assumption of capital agglomeration in explaining increasing returns is consistent with

the observations of reality that the emergence of many metropolitan cities in the world in the twenty-first century are characterised by massive inflow of capital.

Our empirical results, which give modest support to the model, encourage further research along the lines of the model. In particular, the neoclassical model does not predict negative coefficients on the real rental value of capital on population or employment *relative* to that in the nation. Further, although the estimates of the coefficients on time are not inconsistent with the neoclassical system (at the 5% level the null hypothesis cannot be rejected) their *P*-values are large enough to raise some doubt. This is especially true since the signs of the estimated coefficients are positive in both equations. This suggests that data covering a longer span of time, and/or in which the region is defined to be a large metropolitan area, may indicate that the C-model of Guccione and Gillen (1980), which assume constant return to scale, is appropriate only as a first approximation. In other words, the model could be improved by the introduction of a secular drift designed to capture the effects of IRS.

References

Bank of Canada. 1991. *Bank of Canada Review*. Ottawa: Bank of Canada.

Dixon, R. and A. P. Thirlwall. 1975. "A Model of Regional Growth Rate Differences on Kaldorian Lines". *Oxford Economic Paper* 27: 201-214.

Faini, R. 1984. "Increasing Returns, Non-traded Inputs and Regional Development".*Economic Journal* 94: 308-323.

Fujita, M. and P. Krugman. 2004. "The New Economic Geography: Past, Present and the Future". *Papers in Regional Science* 83: 139-164.

Gillen, W. J. and A. Guccione. 1983. "The Relevance of Wage Differentials in Neoclassical Urban Growth". *Regional Science and Urban Economics* 13: 547-555.

Guccione, A. and W. J. Gillen. 1980. "Export-base and Neoclassical Type Model of Urban Growth: A Synthesis". *Canadian Journal of Economics* 13: 700-711.

Kaldor, N. 1970. "The Case for Regional Policies". *Scottish Journal of Political Economy* 17: 337-348.

Krugman, P. 1981. "Trade, Accumulation and Uneven Development". *Journal of Development Economic* 8: 143-162.

Krugman, P. 1991. "Increasing Returns and Economic Geography". *Journal of Political Economy* 99: 483-499.

Krugman, P. 1993. "First Nature, Second Nature, and Metropolitan Location". *Journal of Regional Science* 33: 129-144.

Krugman, P. 1995. *Development, Geography, and Economic Theory*. Cambridge, Massachusetts and London, England: The MIT Press.

Krugman, P. 1998. "Space: The Final Frontier". *Journal of Economic Perspectives* 12:161-174.

Samuelson, P. 1952. "The Transfer problem and Transport Cost, II: The Terms of Trade when Impediments are Absent". *Economic Journal* 62: 278-304.

Samuelson, P. 1983. "Thünen at Two Hundred". *Journal of Economic Literature* 21: 1468-1488.

Statistics Canada. 1991. *Annual Review of Employment and Payrolls.* Ottawa: The Queen's Printer.

Statistics Canada. *Canadian Economic Observer, Historical Statistical Supplement*, 1991/1992. Ottawa: Statistics Canada.

Statistics Canada. 1991. *Historical Statistics of Canada*. Queen's Printer, Ottawa.

Thünen, V-J. H. 1824. *Der Isoliete in Beziehung auf Landwirtschaft und Nationalokonomie.* Third ed. Ed.: Heinrich Waentig, Jena: Gustav Fischer, 1930. (See Carla, M. 1966. *Thünen Johann Heinrich von, 1783-1850 Isolated State,* for an English translation.)

Young, A. 1928. "Increasing Returns and Economic Progress". *The Economic Journal* 38: 527-542.

Appendix 1: Taylor Approximation of the Regression Model

To obtain these two equations it is convenient to use Taylor approximations. They are:

$$Ln(\frac{\overline{P}/P - 1}{1 + \delta - \overline{P}/P}) = LnC_o + C_1 Ln(\overline{P}/P)$$

and

$$Ln(N + \overline{N}) \cong \ln \lambda_0 + (1 - \lambda_1)Ln\overline{N} + \lambda_1 LnN$$

where

$$LnC_0 = Ln(\frac{d_0}{1 - d_0}) - (\frac{1 + \delta d_0}{\delta d_0(1 - d_0)})Ln(1 + \delta d_0),$$

$$C_1 = (\frac{1 + \delta d_0}{\delta d_0(1 - d_0)}),$$

$$Ln\lambda_0 = Ln(\frac{N_0 + \overline{N}_0}{\overline{N}_0}) - (\frac{\overline{N}_0}{N_0 + \overline{N}_0})Ln(1 + \delta d_0),$$

$$\lambda_1 = \frac{N_0}{N_0 + \overline{N}_0},$$

and d_0, N_0 and $\overline{N}_0$ are appropriately chosen points around which the approximation is taken. These approximations convert the model to a system of log-linear equations and the reduced form equations are then easily obtained.