Canadian Journal of Regional Science
Revue canadienne des sciences régionales

# Location and land values: comparing the accuracy and fairness of mass appraisal models

Jamie E.L. Spinney[1], Hanna Maoh[2], and Pavlos Kanaroglou[3]

[1]Department of Geography and Environmental Studies, Saint Mary's University, Halifax, NS, Canada; [2]Department of Civil and Environmental Engineering, University of Windsor, Windsor, Ontario, Canada; [3]School of Geography and Earth Sciences, McMaster University, Hamilton, Ontario, Canada. Address comments to jamie.spinney@smu.ca.

Received 24 April 2013. Accepted 12 December 2014



Spinney, JEL, Maoh, H, & Kanaroglou, P. 2014. Location and land values: comparing the accuracy and fairness of mass appraisal models. *Canadian Journal of Regional Science / Revue canadienne des sciences régionales 37(1/3)*, 19-26.

In contrast to traditional specifications of hedonic price models, which inherently fail to adequately capture the influence of location, the abilities of spatial model specifications to explicitly incorporate the impact of location should improve the accuracy and fairness of urban land value estimates. The objective of this research is to compare the relative performance of ordinary least squares regression with both spatial autoregressive and ordinary Kriging models. The purpose of this comparison is twofold: investigate (i) the relative out-of-sample predictive accuracy; and (ii) each model's respective ability to produce fair land value estimates. Using vacant land sales from Hamilton, Ontario, results indicate that the hedonic price models may provide more accurate estimates of residential urban land values, but spatial interpolation may help promote fairness.

Land values represent the economic health of urban areas and statistical analysis of land values supports research on a variety of social, economic, and land-use planning policies. The importance and merits of land values have been around for centuries, and documented almost two centuries ago "how difficult it is to work out the land rent of any given farm; ... [and] it should not surprise us to find that nearly every such attempt has miserably failed in practice" (von Thünen, 1966, p. 212). These difficulties become more complex when attempting to work out the land rent of any given urban lot considering the special characteristics of land, public policy controls, the many different and competing land uses, and the wide variety of market participants and financing methods. Moreover, the price people pay for property is occasionally ill advised (Skaburskis, 2002). These difficulties contribute to making the valuation of vacant land one of the most difficult aspects of property assessment (Gloudemans, Handel, & Warwa, 2002).

Location is clearly an important factor to consider in real estate research, and "land of different situation will command very different rents" (Douglas, 1936, p. 17). The impact of location still manifests itself in the explicit influence of land's location in space on its value. This *inherent geography* of urban land values affords them unique characteristics, such as spatial autocorrelation and spatial heterogeneity, which have received considerable attention in real estate research (for a recent review see Osland, 2010) and are expected to contribute to "the increased use of advanced spatial methods" in the future (Krause & Bitter, 2012: S19). The problem with failing to sufficiently capture the impact of location is that many well-specified appraisal models will violate their statistical assumptions, which may bias and even invalidate the urban land value estimates. Since the assessed value of real estate is the basis for, among other things, calculating property tax burdens, failing to sufficiently capture the impact of location contributes to social and geographic inequities of the property tax (Harris & Lehman, 2001; Spinney & Kanaroglou, 2012). Consequently, and in addition to the various public and private applications of land price data, it is important to consider the choice of modelling technique used to assess urban land values, because it has economic, planning, and social welfare implications.

The purpose of this research is to explore the inherent geography of urban land values by comparing the traditional ordinary least squares (OLS) regression model with two spatial modeling techniques: (1) spatial autoregressive models, and (2) Kriging. The objective of this comparison primarily concerns each model's relative performance with respect to their (a) predictive accuracy and (b) ability to mitigate geographic inequities (i.e. examining fairness) in the appraisal of residential urban land values within the City of Hamilton, Ontario, Canada. Unlike predictive accuracy, fairness is determined by analysis of sales ratios, which are simply quotients from *market value* divided by *market price*. A number of fairness (uniformity) measures including the coefficient of dispersion (COD), and price-related differential (PRD) values will be used in this study (IAAO, 2007). The spatial autoregressive techniques employed here include spatial lag (SPL) and spatial error (SPE) models.

The remainder of this paper continues with some important definitions and theoretical background information, which is followed by a brief description of the study area and the data used to estimate these models. The models are described in the methods section, followed by a comparison of the performance of each modelling technique and some concluding remarks.

**Study area and data description**

The study area for this research is the amalgamated City of Hamilton, Ontario, Canada. It is located approximately 75 kilometres southwest of the provincial capital of Toronto and had a total population of 693,000 in 2006. Like many other North American cities during the post-World War II period, Hamilton experienced substantial economic development and population growth associated with intense urban development. Within the past few decades, Hamilton has been exposed to suburbanization with *greenfield* development of residential and commercial subdivisions. The suburbanization process has changed the traditional roles of the countryside and the city's downtown (Maoh, Koronios, & Kanaroglou, 2010), with a consequent impact on appreciation and depreciation rates in land prices: with the highest values but the lowest appreciation rates in the city's downtown.

The data used to enable the relative performance comparison of four different model specifications within the City of Hamilton can be categorised into price data and contextual data. The transaction price data were acquired from the Land Registry (i.e. deed transfer) office and included information about the location, date of sale, and the nominal sale price for the population of 87,277 private real estate transactions that occurred in Hamilton, Ontario, between January 1995 and May 2004. Using a spatial decision support system (Spinney, Kanaroglou, & Millward, 2010), a total of 2,524 transactions in vacant land were extracted from the population of private real estate transactions.

Contextual data were acquired to provide independent variables, stratify the vacant land market, and adjust nominal prices. Cadastre (i.e. parcel fabric) data were acquired from Teranet Inc. (http://www.teranet.ca) and were used to derive information about the total area for each parcel of land within the study area. Land use data were acquired from the municipality (http://www.hamilton.ca) and provided information primary land use type (e.g. residential, commercial) for each parcel of land within the study area. Using geographic information system (GIS), the location of municipal water and sewer infrastructure, also acquired from the municipality, was used to determine the parcels within the "serviced" area, while the location of public schools was used to determine the distance to each parcel of land in the study area. Statistics Canada's New Housing Price Index (NHPI) includes independently indexed **L**and Price (NHPI-L) information and monthly NHPI data for the City of Hamilton between 1995 and 2003 were downloaded from the E-STAT website (http://estat.statcan.ca), and were used to provide information about the dynamic land market conditions. Statistics Canada's 2001 census data were also downloaded from the E-STAT website and were used to represent the social and economic attributes affecting urban land values.

**Data processing**

After formatting and concatenating the various datasets, the vacant land sales were stratified into market segments. Market stratification or market segmentation is based on the understanding that different goods will have different markets, whereby consumer preferences and prices are largely diversified (Rapkin, Winnick, & Blank, 1953; Grigsby, 1963; Goodman & Thibodeau, 2003; Wheeler *et al*., 2014). The concept of a housing submarket is based on the appraisal concept of substitution, and the central notion of a submarket is that properties should be close substitutes and not just located in the same neighbourhood (Jones, Leishman, & Watkins, 2005). While market segmentation can be used to delineate relatively homogeneous market segments according to either geographical areas (i.e. neighbourhoods) or the physical use of the property (e.g. residential, commercial), it was used in the current study to select relatively homogeneous non-rural residential land uses within the area serviced by municipal water and sewer. Furthermore, residential lots larger than two and a half acres (approximately 8,094 $m^2$), likely planned for subdivision, were excluded in order to improve constant quality among vacant land prices and to account for diminishing returns to lot size (see Colwell & Sirmans, 1978).

Market segmentation resulted in total of 1,751 transactions of urban, serviced, residential, and vacant land parcels within the study area between 1995 and 2003. To enable comparison of land price information from different time periods (and different micro and macro land market conditions) nominal sale prices were multiplied by NHPI-L values for each year required to bring the price information to real prices that represent land market conditions in 2003. Using real prices, the next processing operation was to remove price outliers.

In order to account for constant quality among sale prices and to eliminate any extreme values, we first computed a spatially continuous surface of mean land prices per square metre using an adaptive kernel. CrimeStat® III software (Levine, 2009) was used to compute an adaptive kernel using 100 m grid cells with a Gaussian functional form and 30 nearest neighbours, and the mean values were extracted to each sale point. This local mean price per square metre was then divided by the *market price* per square metre to compute local ratio values. Similar to Gatzlaff & Ling (1994) only those transactions with local ratio values within three standard deviations of the overall mean were selected: leaving 1,640 transactions in vacant land to enable the comparison of relative accuracy and fairness of OLS, spatial autoregressive, and ordinary Kriging models. Before a description and comparison of the various model specifications and their respective abilities to incorporate the impacts of location into the assessment of urban land values, however, the independent variables used in the hedonic price models are examined.

The traditional mantra used to describe the three main factors affecting the value of real estate is "location, location, and location" (Britton, Davies, & Johnson, 1989; Cohen & Coughlin, 2008). Location may be separated into (i) site factors (e.g. size, shape or configuration, slope, drainage), (ii) situa-

**TABLE** 1. Summary statistics of estimation and validation samples

| | *Estimation sample (n=1,497)* | | *Validation sample (n=143)* | | *Comparison of means* | |
|---|---|---|---|---|---|---|
| *Variable* | *Mean* | *Standard deviation* | *Mean* | *Standard deviation* | *t* | *Sig. ** |
| Real sale price ($) | 74,889 | 52,691 | 73,761 | 36,954 | 0.250 | 0.803 |
| Ln(Real sale price ($)) | 11.10 | 0.45 | 11.12 | 0.41 | -0.512 | 0.609 |
| Parcel area ($m^2$) | 584 | 517 | 546 | 312 | 0.873 | 0.383 |
| Median income ($) | 69,596 | 10,982 | 70,330 | 12,029 | -0.757 | 0.449 |
| Distance to CBD (km) | 8.19 | 3.67 | 8.04 | 3.41 | 0.470 | 0.639 |
| Ln(Distance to CBD km)) | 2.012 | 0.435 | 1.997 | 0.440 | 0.394 | 0.694 |
| Distance to school (km) | 0.648 | 0.393 | 0.611 | 0.490 | 1.052 | 0.293 |

* Two-tailed significance

tion in space factors (i.e. proximity to physical (e.g. highways), legal, social (e.g. schools), land use type (based on zoning), and economic (e.g. Central Business District (CBD)) attributes affecting value), and (iii) situation in time factors. The selection of independent variables was partially informed by theory and previous research, but was also based on results from exploratory data analysis (i.e. correlation analysis and multicollinearity tests). Site factors are represented by parcel area, which was measured in square metres. Situation in space factors were represented by several independent variables:

(i) median income in 2001 by census tract (n = 166);

(ii) straight-line distance to nearest school;

(iii) straight-line distance to (CBD);

(iv) freeway proximity (1 if parcel within 1500 metres, 0 otherwise); and

(v) land uses types, which included two variables: farm land use (1 if the parcel is located on farm land, 0 otherwise) and row-housing land use (1 if the parcel is located on land zoned row-housing, 0 otherwise).

Farm parcels here are within city limits. Finally, situation in time was accounted for by temporally adjusting nominal sale prices into real prices for vacant land (the dependent variable) that reflect 2003 land market conditions.

The comparison of model performance is primarily based on each model's predictive ability, so the 1,640 vacant land transactions were divided into two randomly sampled groups; the result is a relatively large estimation sample (i.e. in-sample observations) and a relatively small validation sample (i.e. out-of-sample observations). The estimation sample used to estimate the different model specifications contains 1,497 observations and the sample used to validate those models has 143 observations. It is important that the comparison of model performance is not inexplicably influenced by a poorly selected validation sample (Case *et al*., 2004; Páez, Long, & Farber, 2008), so an independent samples t-test was used to ensure the absence of any statistically significant differences between the estimation and validation samples (Table 1).

The two samples exhibit similar means and standard deviations. For example, the maximum sale price ranges from $10,000 to over $670,000, yet the difference in mean sale prices is only $1,441. The independent samples t-test results provide further evidence that, despite any apparent differences in mean values, none are significant. Table 1 provides convincing evidence that the estimation and validation samples are reasonably similar over all the dependent and independent variables used to estimate the different models.

## Methods

The purpose of this section is to describe the different models used to estimate residential land values and the methods used to compare their relative performance. Analysis of each model's predictive ability will follow in the results section along with a comparison of the accuracy and fairness of each modelling technique.

As previously mentioned, OLS is the most commonly used parameter estimation method for modeling land values. The OLS model may be represented using matrix notation as

$$\boldsymbol{Y} = \boldsymbol{X}\,\boldsymbol{\beta} + \varepsilon \tag{1}$$

where $\boldsymbol{Y}$ is a (n x 1) vector of observed sale prices on *n* parcels of land; $\boldsymbol{X}$ is a (n x k) vector of site and situation characteristics for parcels of land; $\beta$ is a (k x 1) vector of unknown coefficients; and $\varepsilon$ is a (n x 1) vector of the net effect of all the other factors affecting sale prices but omitted from the model (Bowen *et al*., 2001). Although the observed sale price $\boldsymbol{Y}$ could be utilized in the OLS model, applying a log transformation to the price values (i.e. Ln($\boldsymbol{Y}$)) would help address potential heteroscedasticity and will eliminate the chance of making negative price prediction. As such, the OLS model can be rewritten as

$$\mathrm{Ln}(\boldsymbol{Y}) = \boldsymbol{X}\,\boldsymbol{\beta} + \varepsilon \tag{2}$$

Here, the unknown $\boldsymbol{\beta}$ coefficients are estimated by OLS as

$$\boldsymbol{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\,\mathrm{Ln}(\boldsymbol{Y}) \tag{3}$$

Linear multiple regression of sale prices was initially carried out in SPSS using a simple additive model and served as the benchmark against which the three subsequent models that will be evaluated.

The OLS model assumes independence in the error term $\varepsilon$. However, more often than not spatial autocorrelation is likely to be present in spatial data. Failing to account for spatial autocorrelation in the OLS model will cause the estimated $\boldsymbol{\beta}$ to be inefficient. The presence of spatial autocorrelation in the data can be determined by estimating the Global *Moran's I* sta-

**TABLE** 2. Comparison of regression model parameters

| | *OLS* | | *SPL* | | *SPE* | |
|---|---|---|---|---|---|---|
| | *beta* | *p-value* | *beta* | *p-value* | *beta* | *p-value* |
| Constant | 10.311 | 0.00000 | 5.660 | 0.00000 | 10.171 | 0.00000 |
| Parcel area* | 5.287 | 0.00000 | 4.355 | 0.00000 | 4.659 | 0.00000 |
| Median income* | 0.053 | 0.00000 | 0.017 | 0.01287 | 0.059 | 0.00003 |
| Freeway proximity | 0.029 | 0.03915 | 0.020 | 0.10431 | 0.026 | 0.33710 |
| Farm land use | 0.131 | 0.00016 | 0.091 | 0.00233 | 0.129 | 0.00000 |
| Row-housing land use | -0.606 | 0.00000 | -0.375 | 0.00000 | -0.541 | 0.00000 |
| Ln(Distance to CBD) | 0.043 | 0.02868 | 0.013 | 0.45326 | 0.098 | 0.02023 |
| Distance to school | 0.050 | 0.00832 | 0.006 | 0.70170 | 0.074 | 0.01184 |
| Lag coefficient rho | | | 0.453 | 0.00000 | | |
| Lag coefficient lambda | | | | | 0.676 | 0.00000 |
| R-square | 0.638 | | 0.731 | | 0.766 | |
| N | 1,497 | | 1,497 | | 1,497 | |
| Standard error | 0.271 | | 0.234 | | 0.218 | |
| Akaike info criterion (AIC) | 355.364 | | -27.502 | | -148.458 | |

* Parameter scaled by 10,000

tistic. If the latter is significant then the null hypothesis of no spatial autocorrelation is rejected and the tested variable is said to exhibit spatial autocorrelation. In order to control the impacts of spatial effects, the multiple regression OLS model can be extended into what is known as the spatial lag model. Typically, spatial effects could manifest themselves through the dependent variable Ln($\boldsymbol{Y}$) or the error term $\boldsymbol{\varepsilon}$. If the spatial effects are present in Ln($\boldsymbol{Y}$), then the error term $\boldsymbol{\varepsilon}$ of the OLS equation (eq. 2) is decomposed into a spatially lagged term $\rho\boldsymbol{W}\text{Ln}(\boldsymbol{Y})$ (calculated as a weighted average of neighbouring values Ln($\boldsymbol{Y}$)) and an independent error term $\epsilon$. Here, $\rho\boldsymbol{W}\text{Ln}(\boldsymbol{Y})$ is correlated with the dependent variable Ln($\boldsymbol{Y}$). This treatment to the OLS yields the spatial lag (SPL) model, which takes the following form:

$$\text{Ln}(\boldsymbol{Y}) = \boldsymbol{X\beta} + \rho\boldsymbol{W}\text{Ln}(\boldsymbol{Y}) + \boldsymbol{\epsilon} \quad (4)$$

Here $\rho$ is the spatial lag parameter and $\boldsymbol{W}$ is the (n x n) neighbourhood matrix of spatial dependence. All other symbols are as in the OLS model. The spatial autocorrelation term $\rho\boldsymbol{W}\text{Ln}(\boldsymbol{Y})$ is added to the linear regression model in order to capture the strength of the spatial dependence among the observations of the dependent variable Ln($\boldsymbol{Y}$). We created a Thiessen polygon layer from the point representations of the parcels and derived a first order rook contiguity matrix W from these polygons. The rows of the neighbourhood matrix $\boldsymbol{W}$ sum to 1, which means that $\boldsymbol{W}$ is row-standardized. On the other hand, if the spatial effects are manifested through the error term $\varepsilon$ itself, then this term can be written as the sum of a spatial dependent term $\lambda\boldsymbol{W}\varepsilon$, which captures the spatial autocorrelation between the neighbouring error terms $\boldsymbol{\varepsilon}$, and an independent error term $\boldsymbol{\epsilon}$. Such treatment gives rise to the spatial error model (SPE), which takes the following form:

$$\text{Ln}(\boldsymbol{Y}) = \boldsymbol{X\beta} + \lambda W\varepsilon + \boldsymbol{\epsilon} \quad (5)$$

$\lambda$ here is the spatial error parameter. GeoDa™ software (ver. 1.6.5 11) was used to estimate the spatial models using the Maximum Likelihood method and asymptotic inference (see Smirnov & Anselin, 2001).

Kriging predicts the value of a variable at a point in space on the basis of observed values for the variable. Observations closer to the prediction point are assigned higher weights than those further away. Kriging is based on the assumption that the variable being interpolated can be treated as a regionalized variable, meaning it is spread out in space and/or time (Krige, 1951; Matheron, 1963). There are relatively few applications of Kriging in real estate research (e.g. Dubin, 1998; Des Rosiers *et al.* 2001; Case *et al.*, 2004; Chica-Olmo, 2007; Páez, Long, & Farber, 2008) and even fewer applications of Kriging models to land prices (e.g. Shultz, 2007; Tsutsumi, Shimada, & Murakami, 2011; Hu *et al*., 2013).

Recall that the impact of location on land values may be separated into absolute location in space, relative location in space, and relative location in time. It is possible to incorporate the absolute location in space (i.e. parcel area) into the dependent variable by using sale price per square metre. It is important to note that this specification of the dependent variable assumes the price of land is directly proportional to the size of the lot. Meanwhile, the relative location in time has already been incorporated into the dependent variable by using the NHPI-L to temporally adjust nominal prices of historical transactions into real land prices that represent 2003 land market conditions. The remaining impact of location is the relative location in space, which is embodied in the sale price, and is thus captured by the Kriging model.

Based on analysis of trends and the presence of local stationarity exhibited in the variograms (covariance models), we chose ordinary Kriging with local variograms (see Haas, 1990) for spatial prediction of urban land values. The software used to perform Kriging with local variograms is called VESPER, which is an acronym for Variogram Estimation and Spatial Prediction with Error, and was developed by the Australian Centre for Precision Agriculture (Minasny, McBratney, & Whelan, 2005). The advantage of fitting of a local variogram model stems from the ability of the Kriging model to adapt to differences in local spatial structure over the study area, which

**TABLE** 3. Summary statistics and comparison of model prediction performance

| *Model* | *Mean absolute error* | *Median absolute error* | $R^2$ | *Predictions within 10% of validation price* | *Predictions within 20% of validation price* | *Predictions within 30% of validation price* |
|---|---|---|---|---|---|---|
| OLS | 14,424.0 | 8,821.8 | 0.775 | 32.9 | 67.8 | 83.9 |
| SPL | 12,985.6 | 8,215.4 | 0.815 | 45.5 | 70.6 | 83.9 |
| SPE | 11,946.6 | 7,019.0 | 0.793 | 49.7 | 74.1 | 86.0 |
| Kriging | 15,169.5 | 11,047.8 | 0.769 | 32.9 | 58.7 | 76.9 |

should produce more accurate predictions than a global variogram.

**Model evaluation**

Evaluating the relative performance of models begins with a comparison of parameter estimates for the OLS and spatial autoregressive (i.e. SPL and SPE) models, followed by an evaluation of the predictive accuracy of each model, including the Kriging model. Predictive accuracy is assessed by comparing predicted values with the observed values in the validation sample. The predictive accuracy of each model specification is also evaluated using sales ratios, which provide a statistical measure of how close the *market value* is to *market price*. Market price is the amount actually paid in a particular transaction, while market value is a hypothetical or estimated sale price that would result from careful consideration by the buyer and seller of all data, with primary reliance on those data that reflect the actions of responsible, prudent buyers and sellers under conditions of a fair sale. Standard sales ratio study metrics are used to evaluate the accuracy and fairness of the land value estimates from the three different model specifications.

The purpose of this section is to compare the results of the benchmark OLS model with the spatial autoregressive and Kriging models. It is important to reiterate that the objective of this research is to compare the relative performance of four statistical models. The comparison first examines the model parameters then examines the performance of the different model specifications in terms of out-of-sample predictive accuracy.

A comparison of model parameters for the OLS and maximum likelihood SPL and SPE parameter estimation methods is provided in Table 2, and illustrates relatively stable coefficients for the independent variables used to explain vacant land prices.

We looked for any evidence of multicollinearity among our independent variables. The correlation between most pairs of variables was weak; under 0.20. The only two variables that showed some affiliation were Ln[Distance to CBD] and Median income; correlation 0.52. However, all the estimated coefficients meet our a priori expectation in terms of their expected signs. According to the OLS model in Table 2, larger land parcels have higher values, other things being equal. Also, parcels within 1500 meters from a freeway are more valuable compared to parcels outside that distance range. Likewise, parcels in areas with higher median income have higher values. In terms of the effect of land use type, parcels located on farmland use are more valuable while parcels located on land designated for row houses are less valuable, other things being equal. As for proximity measures, the estimates suggest that parcels in locations far away from CBD or schools are more valuable.

While all coefficients are significant in the OLS model, the coefficients for distance to school and distance to CBD (i.e. Ln(Distance to CBD))have become insignificant in the SPL model, which illustrates the effect that spatial autocorrelation can have on OLS estimates (see Anselin, 2004). Likewise, the coefficient for Freeway Proximity has lost its significance in the SPE model. The R-squared values, although not directly comparable due to the manner in which they are calculated, represent the proportion of the variation of vacant land sale prices that is accounted for by each model and are remarkably similar, especially for the spatial autoregressive models, and relatively high. The standard error of the estimate indicates the extent to which the estimated sale prices vary from their actual values, and the values are remarkably similar across the three models, but slightly improved for the SPE model. In a similar vein, the SPE model has the lowest AIC value among the three regression models suggesting that it is the best model in terms of the trade-off between its goodness-of-fit and complexity.

Before running the autoregressive models, spatial autocorrelation tests via the Global Moran's *I* statistic were performed in GeoDa. Moran's *I statistic* for Ln($\boldsymbol{Y}$) was estimated to the value of 0.562 (p-value = 0.001, z-value = 36.51), which suggests the presence of spatial autocorrelation in Ln($\boldsymbol{Y}$). Likewise, Moran's *I statistic* for the errors $\varepsilon$ of the OLS model shown in eq. 2 was also estimated to the value of 0.400 (p-value = 0.001, z-value = 25.68). which is also indicative of spatial autocorrelation in the obtained OLS errors. The existence of spatial autocorrelation in Ln($\boldsymbol{Y}$) and $\varepsilon$ justify the use of spatial autoregressive models to control for any potential estimation bias in the OLS coefficients. The significance of the lag parameters $\rho$ and $\lambda$ in the SPL and SPE models is indicative that the effects of spatial autocorrelation in the dependent variable Ln($\boldsymbol{Y}$) and the errors $\varepsilon$ have been controlled for in the two models, respectively. An examination of the estimated coefficients indicates that the use of the spatial lag term $\rho\boldsymbol{W}$Ln($Y$) in the SPL model helped reduce the bias in the estimated OLS coefficients. Consequently, the SPL coefficients are all smaller in terms of their magnitude when compared to the OLS coefficients. The same could be said about

**TABLE** 4. Comparison of sales ratios and fairness

| *Statistic* | *OLS* | *SPL* | *SPE* | *Kriging* |
|---|---|---|---|---|
| Count of observations | 1,497 | 1,497 | 1,497 | 1,497 |
| Total appraised value | 110,693,492 | 110,411,642 | 107,431,640 | 118,458,760 |
| Total sale price | 112,109,180 | 112,109,180 | 112,109,180 | 112,109,180 |
| Mean appraised value | 73,944 | 73,755 | 71,765 | 79,131 |
| Mean sale price | 74,889 | 74,889 | 74,889 | 74,889 |
| Mean ratio | 1.041 | 1.038 | 1.032 | 1.072 |
| Median ratio | 1.010 | 0.992 | 1.005 | 0.959 |
| Weighted mean ratio | 0.987 | 0.985 | 0.958 | 1.057 |
| Price-related differential (PRD) | 1.054 | 1.054 | 1.077 | 1.015 |
| Coefficient of dispersion (COD) | 0.200 | 0.198 | 0.202 | 0.251 |

the coefficients of the SPE model.

**Comparison of predictive accuracy**

The performance of these models is illustrated in Table 3 using summary statistics and comparative analysis of predictive accuracy that are based on the difference between predicted values from the estimation sample and the observed values of the validation sample.

The mean absolute error (MAE) indicates an increase in model performance for the SPE model over the other models, but the Kriging model has the highest MAE. Interestingly, the OLS model has a relatively high MAE compared to the SPE and SPL models, placing it close to the Kriging MAE value. Since the median is less affected by extreme values, the International Association of Assessing Officers (IAAO) generally prefer the median as the measure of central tendency for monitoring appraisal performance. The median absolute error exhibits a similar pattern as the mean absolute error, with SPE exhibiting the lowest median absolute error. However, the OLS median absolute error became relatively smaller when compared to the SPL and SPE models. The $R^2$ value represents the squared Pearson correlation coefficients between the predicted and observed sale prices in the validation sample. The SPL model has the highest $R^2$, while the OLS and Kriging models are only marginally inferior. The SPE model has a very similar $R^2$ like the SPL.

The last three columns in Table 3 represent the proportion of estimated sale prices that are within 10, 20, and 30 percent of the observed sale prices in the validation sample. For example, 49.7 percent of the prices estimated using the SPE model are within 10 percent of the observed sale prices in the validation sample, compared to only32.9 percent for the OLS and Kriging models, and 45.5 percent for the SPL model. Overall, the Kriging model has the least predictive accuracy at all levels. However, the predictive accuracy of the SPE model retains its superiority when compared to the OLS model within the 20 and 30 percent of the observed sale prices, respectively. This is partly due to the reduction in estimation bias through the spatial lag parameter in the SPE model. The predictive accuracy of the SPL model is superior to the OLS model but not as remarkable as the SPE model. Typically, in the presence of strong spatial autocorrelation, it is likely that the spatial regression model will significantly outperform the OLS model, which is the case as shown in Table 3.

**Comparison of fairness**

Fairness is determined by analysis of sales ratios, which are simply quotients from *market value* divided by *market price*, using the estimation sample presented in Table 4. The desired sales ratio is 1.00, which means the mass appraisal model was able to accurately predict the within-sample prices. However, a sales ratio of 1.00 is unlikely, so the 2007 Standard on Ratio Studies set by the International Association of Assessing Officers (IAAO) indicate that a sales ratio between 0.90 and 1.10 are considered acceptable. We used assessment ratios, coefficient of dispersion (COD), and price-related differential (PRD) values to evaluate each model's respective ability to produce fair estimates of market value.

According to the mean, median, and weighted mean ratios listed in Table 4, all models generated estimates of vacant land values that are considered "acceptable" by IAAO standards. However, the overall ratios do not provide any indication of uniformity or fairness. The most important measure of assessment uniformity is the COD, which represents the average percentage deviation from the median ratio and can be loosely interpreted as the average error, but it does not depend on the assumption that the ratios are normally distributed. According to IAAO standards, COD values for vacant land should not exceed 20.0 percent. COD is calculated by dividing the average of the "absolute deviation of ratios about the median" by the median ratio. Both the OLS and SPE models are close to meeting IAAO standards for COD while the SPL is below the defined threshold. Another measure of uniformity is the price-related differential (PRD), which is used to measure uniformity between high- and low-value properties and should be between 0.98 and 1.03 to demonstrate vertical equity (IAAO, 2007). The PRD is calculated by dividing the mean ratio by the weighted mean ratio. According to the results in Table 4, only the Kriging model was able to produce estimates of vacant land prices that meet IAAO standards for PRD, which suggests that the Kriging model is better able to incorporate the differences between high and low value land parcels.

**Conclusion**

Land value information is necessary in the private sector for lending and investment decisions, and is required in the public sector for land use zoning, eminent domain, and, of course, property taxes. The objective of this research was to compare the relative performance of OLS, spatial autoregressive, and ordinary Kriging models insofar as the accuracy and fairness of the estimates of land values produced. The intention was to compare simple model specifications in order to focus on their respective ability to produce accurate and fair estimates of land values, primarily as a function of their ability to incorporate the impact of location. This research is not, however, without its limitations. A simple linear specification was chosen for the functional form of the OLS and spatial autoregressive models even though we recognise the relationships are likely more complex. We also included sales data over a nine-year period; although nominal prices were adjusted to real prices representing land market conditions in 2003, markets change and neighbourhoods appreciate and depreciate at different rates within the city.

Despite the limitations, we contend that the Kriging model performed very well, especially considering its specification did not incorporate any neighbourhood attributes. However, results clearly indicate that multivariate regressions have significant potential to outperform spatial interpolation of urban land values, and there appears to be convincing evidence for spatial error (SPE) models to improve the accuracy of hedonic price models (Table 3) when specified properly. On the other hand, insofar as each model's respective ability to account for differences between high and low-value lots, only the estimates from the Kriging model meet IAAO standards for vertical equity (Table 4).

Despite having the poorest predictive accuracy of the models tested, the Kriging model highlighted the advantages of explicitly incorporating local spatial dependence and spatial heterogeneity into the model structure, especially when the dependent variable contains measurement errors. Furthermore, the specification of the Kriging model is hampered by the specification of the dependent variable, because the relationship between price and area is almost certainly not linear. A better specification of the dependent variable in the Kriging model, such as a different specification between price and area, or possibly using price per street frontage, could improve the overall performance of the Kriging model, especially in areas with highly variable lot depths. Furthermore, it is possible to incorporate a covariate into the Kriging model by using cokriging, which would invariably improve model performance. Overall, however, these results suggest that perhaps other spatial analytic techniques need to be adopted, such as generalised least squares or geographically weighted regression, that can take advantage of both the spatial distribution of land prices plus the ability to decompose vacant land values into marginal prices.

## References


Anselin, L. 2004. *GeoDa™ 0.9.5-i Release Notes*. Retrieved April 23, 2013 from the Spatial Analysis Laboratory, Department of Agricultural and Consumer Economics, University of Illinois, Urbana-Champaign and Center for Spatially Integrated Social Science. http://geodacenter.org/downloads/pdfs/geoda095i.pdf

Britton, W, Davies, K, & Johnson, T. 1989. *Modern Methods of Valuation*. London, England: Estates Gazette.

Case, B, *et al*. 2004. Modeling spatial and temporal house price patterns: a comparison of four models. *Journal of Real Estate Finance and Economics* 29, 167-191.

Chica-Olmo, J. 2007. Prediction of housing location price by a multivariate spatial method: cokriging. *The Journal of Real Estate Research* 29, 91-114.

Cohen, JP, & Coughlin, CC. 2008. Spatial hedonic models of airport noise, proximity, and housing prices. *Journal of Regional Science* 48, 859-878.

Colwell, PF, & Sirmans, CF. 1978. Area, time, centrality and the value of Urban Land. *Land Economics* 54, 514-519.

Des Rosiers, F, *et al*. 2001. *Isolating Spatial from A - spatial Components of Housing Attributes Using Kriging Techniques*. Working Paper # 2001-008, Faculty of Business Administration, Laval University. http://eres.scix.net/data/works/att/eres2001_149.content.pdf

Douglas, FCR. 1936. *Land Value Rating: Theory and Practice*. London, England: Leonard and Virginia Woolf at the Hogarth Press.

Dubin, RA. 1998. Predicting house prices using multiple listings data. *Journal of Real Estate Finance and Economics* 17, 35-59.

Gatzlaff, DH, & Ling, DC. 1994. Measuring changes in local house prices: an empirical investigation of alternative methodologies. *Journal of Urban Economics 35*, 221-244.

Gloudemans, RJ, Handel, S, & Warwa, M. 2002. *An Empirical Evaluation of Alternative Land Valuation Models*. Boston, MA: Lincoln Institute for Land Policy.

Goodman, AC, & Thibodeau, TG. 2003. Housing market segmentation and hedonic prediction accuracy. *Journal of Housing Economics 12(3)*, 181-201.

Grigsby, WG. 1963. *Housing Markets and Public Policy*. Philadelphia, PA: University of Pennsylvania Press.

Haas, TC. 1990. Kriging and automated variogram modeling within a moving window. *Atmospheric Environment: Part A 24*, 1759-1769.

Harris, R, & Lehman, M. 2001. Social and geographic inequities in the residential property tax: a review and case study. *Environment and Planning A 33*, 881-900.

Hu, S, *et al*. 2013. Modeling land price distribution using multifractal IDW interpolation and fractal filtering method. *Landscape and Urban Planning 110*, 25-35.

IAAO. 2007. *Standard on Ratio Studies*. Kansas City, MO: International Association of Assessing Officers.

Jones, C, Leishman, C, & Watkins, C. 2005. Housing market processes, urban housing submarkets and planning policy. *Town Planning Review 76*, 215-233.

Krause, AL, & Bitter, C. 2012. Spatial econometrics, land values and sustainability: trends in real estate valuation research. *Cities* I, S19–S25.

Krige, DG. 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand. *Journal of Chemical, Metallurgical and Mining Engineering Society of South Africa* 52, 119-139.

Levine, N. 2009. *CrimeStat: A Spatial Statistics Program for the Analysis of Crime Incident Locations (v 3.2)*. Houston, TX: Ned Levine & Associates, and Washington, DC: National Institute of Justice.

Maoh, H, Koronios, M, & Kanaroglou, P. 2010. Exploring the land development process and its impact on urban form in Hamilton, Ontario. *Canadian Geographer* 54, 68-86.

Matheron, G, 1963. Principles of geostatistics. *Economic Geology 58*, 1246-1266.

Minasny, B, McBratney, AB, and Whelan, BM. 2005. *VESPER Version 1.62*. Retrieved April 23, 2013 from the Australian Centre for Precision Agriculture, University of Sydney, NSW http://sydney.edu.au/agriculture/pal/software/vesper.shtml

Osland, L. 2010. An application of spatial econometrics in relation to hedonic house price modeling. *Journal of Real Estate Research 32*, 289–320.

Páez, A, Long, F, & Farber, S. 2008. Moving window approaches for hedonic price estimation: an empirical comparison of modeling techniques. *Urban Studies* I, 1565-1581.

Rapkin, C, Winnick, L, & Blank, D. 1953. *Housing Market Analysis*. Washington, DC: US Housing and Home Finance Agency.

Shultz, S. 2007. Differences between agricultural land value surveys and market sales. *The Appraisal Journal 75*, 194-203.

Skaburskis, A. 2002. Generational differences and future housing markets. *Canadian Journal of Regional Science 25*, 353-376.

Smirnov, O, & Anselin, L. 2001. Fast maximum likelihood estimation of very large spatial autoregressive models: a characteristic polynomial approach. *Computational Statistics and Data Analysis 35*. 301-319.

Spinney, JEL, & Kanaroglou, P. 2012. Municipal taxation and social exclusion: examining the spatial implications of taxing land instead of capital. *Canadian Journal of Urban Research 21*, 1-23.

Spinney, JEL, Kanaroglou P, & Millward, H. 2010. Improving access to land price data: a spatial decision support system for cleansing land registry data. *Canadian Journal of Regional Science* 33(2), 57-72.

Tsutsumi, M, Shimada, A, and Murakami, D. 2011. Land price maps of Tokyo metropolitan area. *Procedia Social and Behavioral Sciences 21*, 193–202

von Thünen, JH. 1966. *The Isolated State and Its Relation to Agriculture and the National Economy*. P. Hall, editor, Oxford, NY: Pergamon Press.

Wheeler, DC, *et al*. 2014. A Bayesian approach to hedonic price analysis. *Papers in Regional Science 93(3)*, 663-684.